Gravity Bayesian inversion with second-order smoothness prior over the Delaunay Tessellation

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1 Background

Smoothness prior is usually used in geophysical Bayesian inversion to constrain the fluctuations of model parameters in temporal or spatial variation. We now consider the spatial smoothing of model parameters and use the smooth function in space to fit the model parameters. B-spline function shows its good smoothness on regular grid nodes, but it needs to grid the observed data first. The smoothing method based on Delaunay Tessellation (DT) shows its useful properties that it can be used for irregularly distributed observed data and strongly determined by the density of the observed points distribution. In the past, most smoothness constraints over DT were only first-order, but for the needs of inversion in most cases, second-order smoothness constraints are needed. This poster uses a new method of directly constraining second-order smoothness prior over the 1-layer DT, and compares it with a previous smoothing technique over the 2-layers DT, with an application on gravity Bayesian inversion. Although the DT also has some disadvantages such that large computation time are required to obtain the triangulation, applications with large amounts of data will not be considered for the time being.

2 Problem

The gravity inversion example we use is estimation of near-surface density $\boldsymbol{\rho}$ and smooth Bouguer gravity anomaly $\boldsymbol{B}$, given observed Free-air gravity anomaly $\boldsymbol{F}$. The observation function is

$$\boldsymbol{F} = \mathbf{H}\,\boldsymbol{\rho} + \mathbf{C}\,\boldsymbol{B} + \boldsymbol{\varepsilon}, \tag{1}$$

where $\boldsymbol{\varepsilon}$ represents the uncorrelated Gaussian noise with zero mean and unknown variance in $\boldsymbol{F}$. $\mathbf{H}$ and $\mathbf{C}$ can be obtained by physical principles, but it is not described in detail here.

To get the estimated solutions $\hat{\boldsymbol{\rho}}$ and $\hat{\boldsymbol{B}}$, we solve this inversion problem in Bayesian framework. Likelihood is given by assuming that $\boldsymbol{F}$ follows multivariate normal distribution, $\boldsymbol{F} \sim \mathcal{N}(\mathbf{H}\,\boldsymbol{\rho} + \mathbf{C}\,\boldsymbol{B}, \mathbf{\Sigma}_{\mathrm{d}})$, all diagonal elements of $\mathbf{\Sigma}_{\mathrm{d}}$ are an unknown σ_{d}^2. Prior distribution is given by assuming that the Bouguer gravity anomaly is a smooth function $f\,(x\,,y)$ of space and constraining the flatness and smoothness through a penalty function $\Phi\,(\boldsymbol{B})$, which is the quadratic form of $\boldsymbol{B}$,

$$\Phi\,(\boldsymbol{B}) = \iint \left\{ \left[\frac{\partial f\,(x\,,y)}{\partial x}\right]^2 + \left[\frac{\partial f\,(x\,,y)}{\partial y}\right]^2 \right\} + \left\{ \left[\frac{\partial^2 f\,(x\,,y)}{\partial x^2}\right]^2 + 2\left[\frac{\partial^2 f\,(x\,,y)}{\partial x\,\partial y}\right]^2 + \left[\frac{\partial^2 f\,(x\,,y)}{\partial y^2}\right]^2 \right\} \mathrm{d}x\,\mathrm{d}y. \tag{2}$$

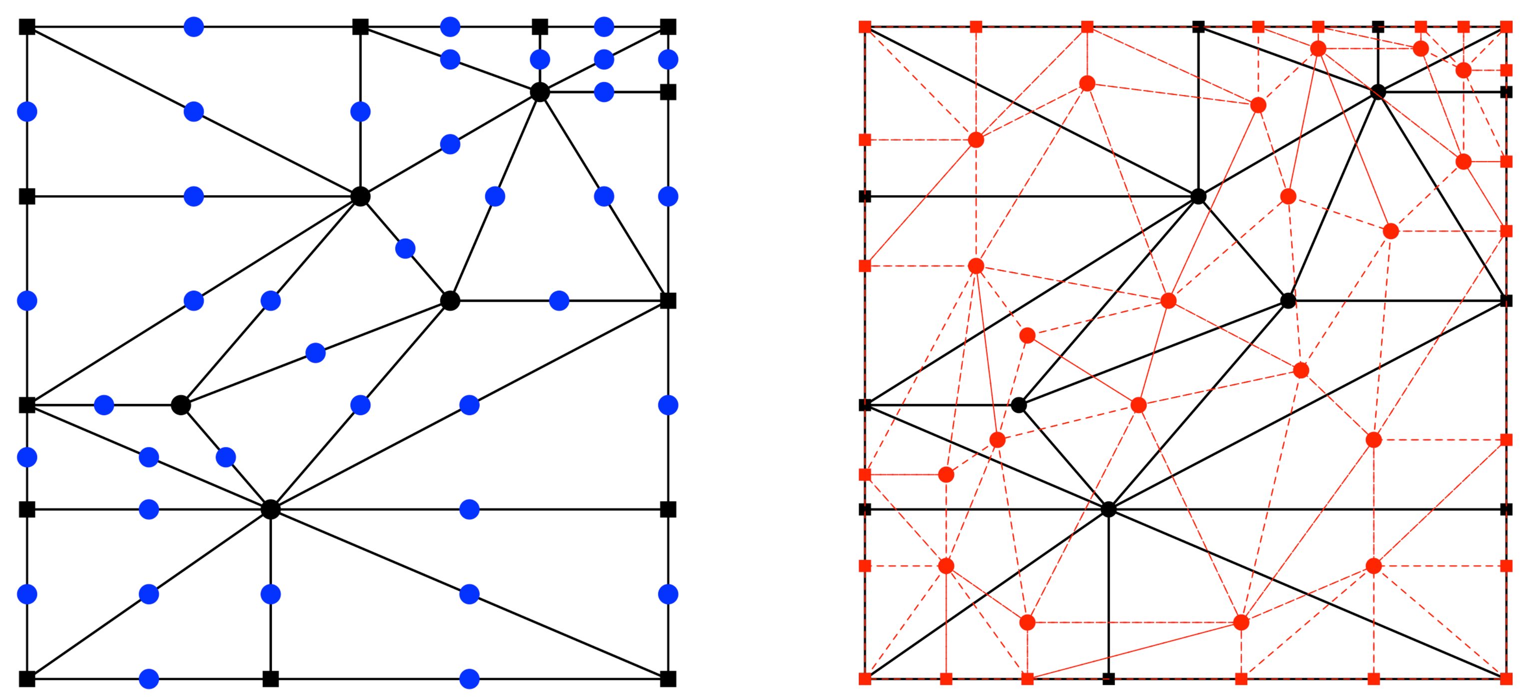
Then the prior of $\boldsymbol{B}$ obeys the multivariate normal distribution, $\boldsymbol{B} \sim \mathcal{N}(\mathbf{0},\, \mathbf{\Sigma})$, and

$$\mathbf{\Sigma} = \left(\frac{\mathbf{D}_1^{\mathrm{T}}\mathbf{D}_1}{\sigma_1^2} + \frac{\mathbf{D}_2^{\mathrm{T}}\mathbf{D}_2}{\sigma_2^2}\right)^{-1},$$

where the $\mathbf{D}_1$ is defined as flatness constrain matrix, $\mathbf{D}_2$ is smoothness constrain matrix, σ_1^2 and σ_2^2 are unknown variances.

Two hyperparameters $\lambda_1 = \frac{\sigma_d^2}{\sigma_1^2}$ and $\lambda_2 = \frac{\sigma_d^2}{\sigma_2^2}$ are determined by ABIC criterion. Once they are determined, $\hat{\boldsymbol{\rho}}$ and $\hat{\boldsymbol{B}}$ can be obtained by MAP estimation.

3 Two kinds of second-order smoothness constraint



M1: $f\,(x,y) = {\theta_1}^{(m)}x^2 + 2{\theta_2}^{(m)}xy + {\theta_3}^{(m)}y^2 + 2{\theta_4}^{(m)}x + 2{\theta_5}^{(m)}y + {\theta_6}^{(m)}$

M2: $f\,(x,y) = a_1^{(m)}B_1^{(m)} + a_2^{(m)}B_2^{(m)} + a_3^{(m)}B_3^{(m)}$

4 Data and result

The research area is located in the southwestern China, from 99°E to 104°E in longitude and 25°N to 27.5°N in latitude. The Free-air gravity anomaly data comes from the World Gravity Model WGM2012, which is derived from the available earth global gravity models EGM2008. We randomly extracted 500 observation points to simulate the irregular distribution of observed points. Moreover, topographic data used to calculate the terrain correction coefficient with per density is extracted from the ETOPO1 model.

The estimated noise standard deviation in observed Free-air gravity anomaly are closely similar when two kinds of second-order smoothness constraint are used. The estimated solutions are also approximately similar.