Randomized Methods for Nonnegative Constrained Graph Regularized Tensor Network Nonconvex Optimization

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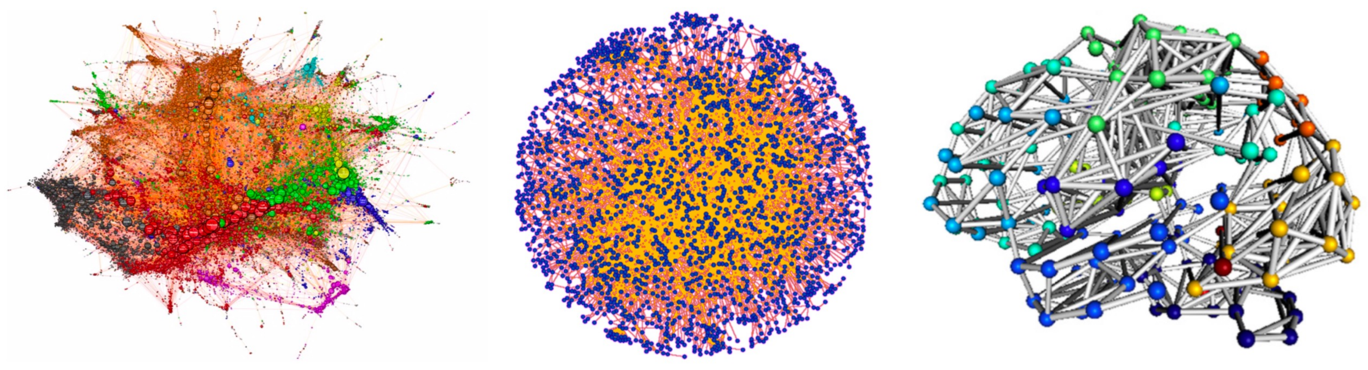
Goals and Challenges

- Real application problems in signal processing and machine learning generate multidimensional data with high dimensionality structures;
- Tensor decompositions aim to represent a higher-order (or multi-dimensional) data as a multilinear product of several latent factors;
- Based on low rank approximation which avoids the "curse of dimensionality";
- Constrained tensor decomposition is a powerful tool for the extraction of parts-based and physically meaningful latent components while preserving multilinear structure;
- From the viewpoint of optimization, the objective function is a nonconvex, nonsmooth with a fidelity term and a regularization term;

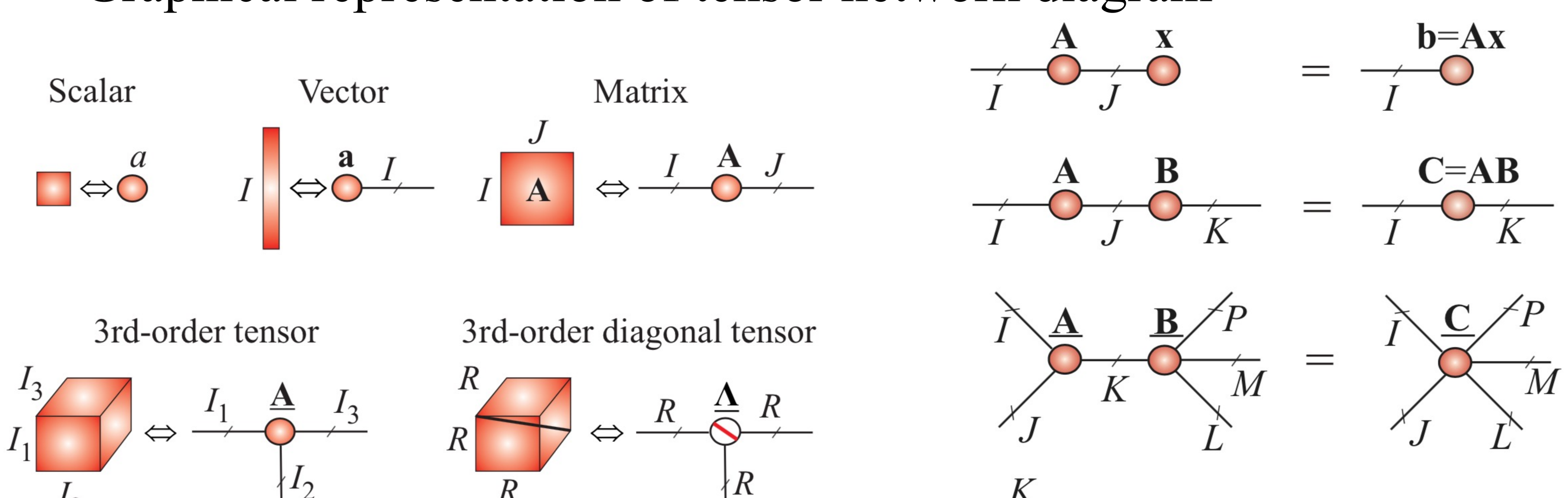


Internet, Biological Networks, Brain Graphs

- Robust and accurate numerical methods are required since the problem is ill-posed, and the numerical solutions are sensitive to the perturbation of input data.

Tensor Networks

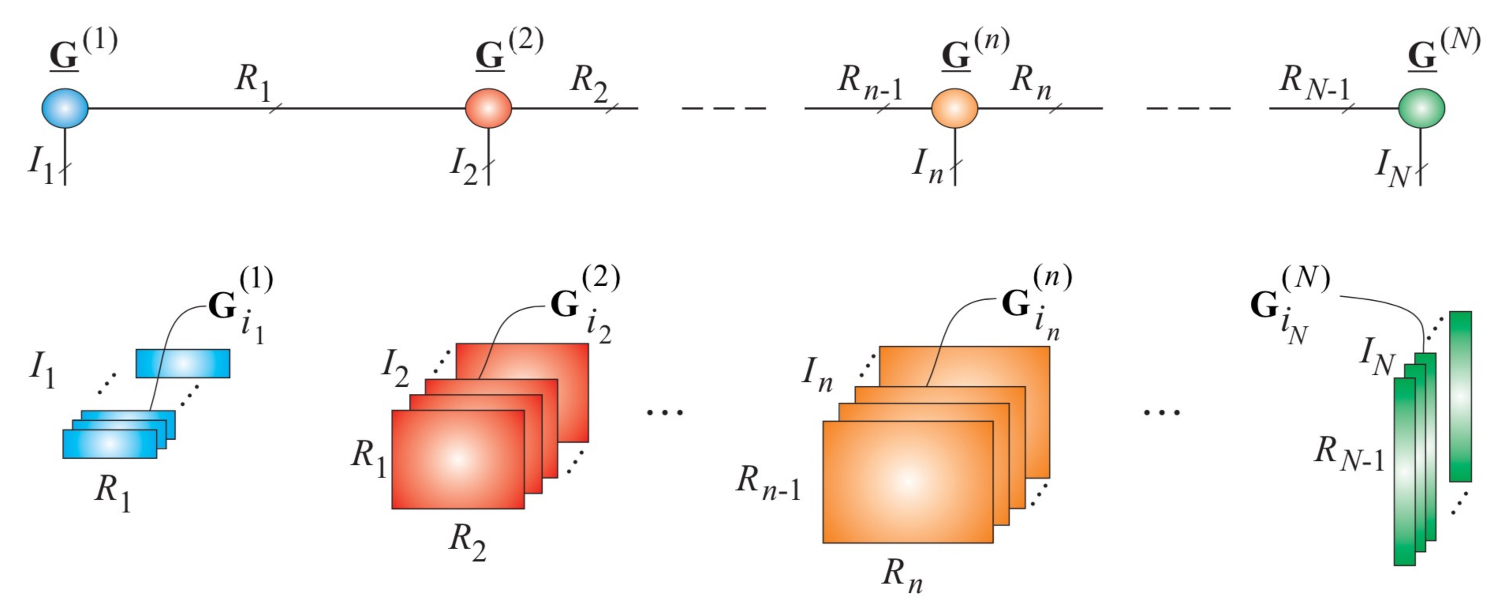
- Representation ability: a powerful tool to describe strongly entangled quantum many-body systems in physics;
- Dimensional / Model reduction: decompose a high-order tensor into a collection of low-order tensors connected according to a network pattern;
- Graphical representation of tensor network diagram



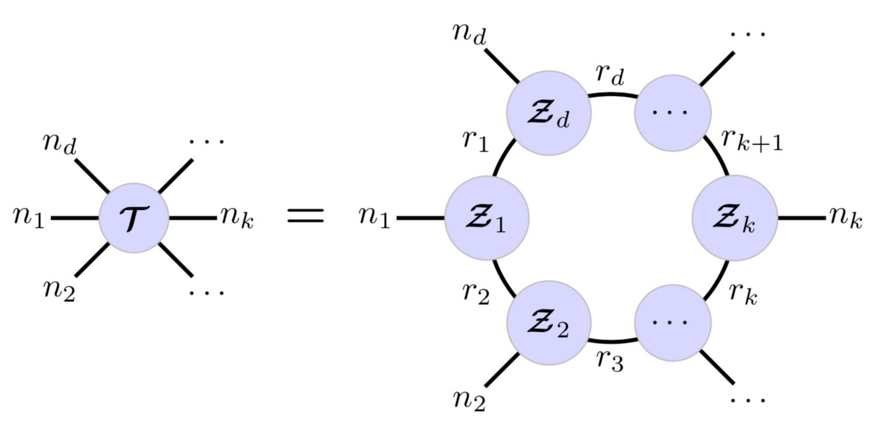
$$\sum_{k=1}^{K} a_{i,j,k}\, b_{k,l,m,p} = c_{i,j,l,m,p}$$

Tensor Train / Ring and Matrix Product State

- Tensor train (TT) decomposition [Oseledets SIAM, 2011]:



- Tensor chain / ring (TR) decomposition: [Zhao, 2018]
- Sum of TT with shared core tensors
- Hierarchical Tucker decomposition



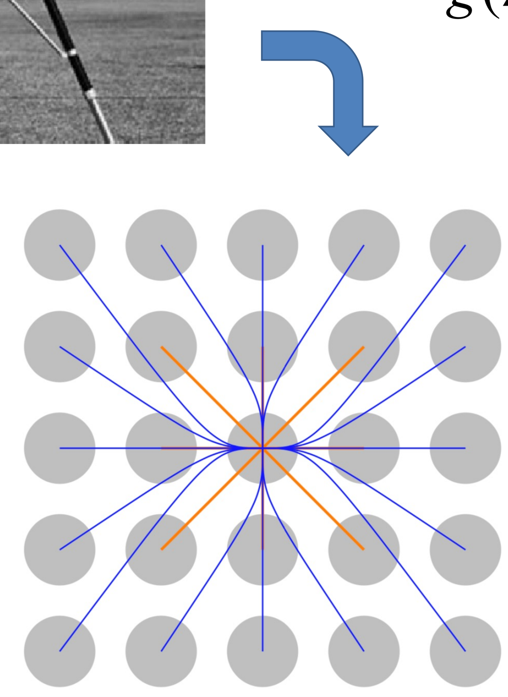
Tensor Networks Nonconvex Optimization

- Given a d-th order tensor, compute the cores with given TR-ranks

$$\min \left\| T - R(Z_1, Z_2, \ldots, Z_d) \right\|_F + \mu g(Z_k)$$
$$s.t. \quad Z_1, Z_2, \ldots, Z_d \in M$$

- Fidelity term denotes the low rank approximation of given tensor;
- Regularization term denotes the prior knowledge for the output core tensors, which are sparse, nonnegative/box constrained, orthogonal (robust PCA), graph structures, etc.
- Alternating least squares method (block coordinate descent method, or block Gauss-Seidel method):
- ■ Alternatively update one core tensor and fix all the other cores tensors
- ■ Solve the subproblem: mode-k unfolding matrix representation:

$$\min \left\| \mathbf{T}_{[k]} - \mathbf{Z}_{k(2)} (\mathbf{Z}_{[2]}^{\neq k})^T \right\|_F + \mu g(\mathbf{Z}_{k(2)})$$
$$s.t. \quad \mathbf{Z}_{k(2)} \in M$$



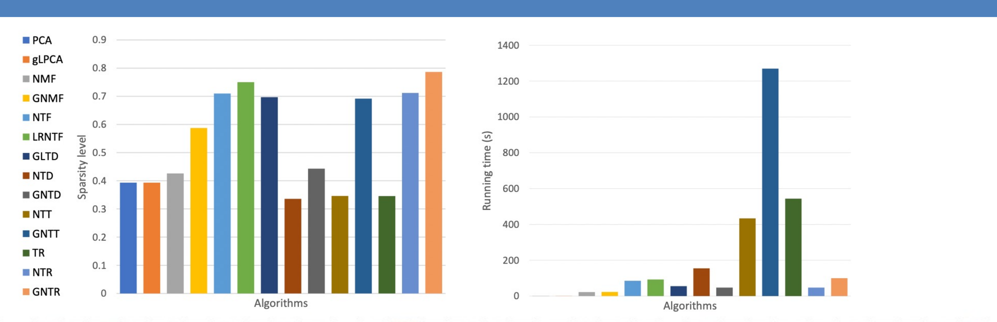
- Task 1: graph topology, geometrical information of data can be obtained by modeling a neighbor graph;

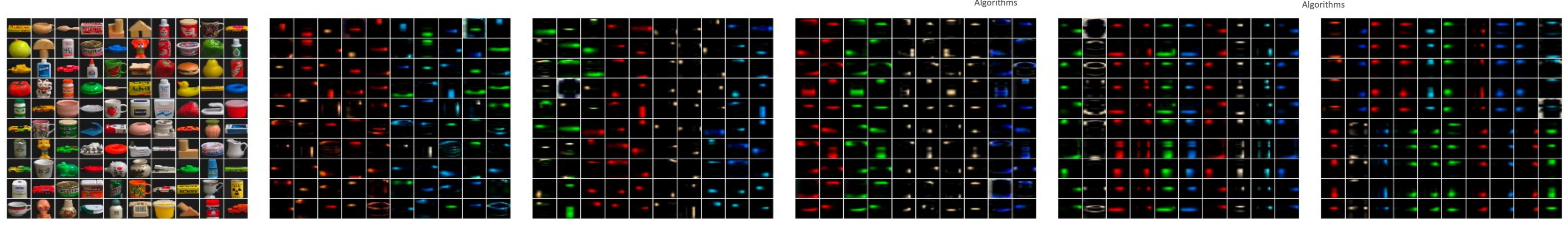
$$g(\mathbf{Z}) = J_{\text{graph}} = \sum_{i,j} (z_i - z_j)^2 W_{ij} = \mathrm{Tr}(\mathbf{Z}^T (D - W)\mathbf{Z})$$

- Task 2: nonnegative tensor network optimization. Other related models: nonnegative matrix factorization (NMF), NCP, NTD, NTT, etc.
- Task 3: rank-selection randomized greedy block coordinate descent iterative algorithm. Solve the subproblem via randomized iterations.

Numerical Experiments

- ORL Database: face images;
- COIL-100 Database;
- Faces PART 1 Database.





Algorithms	Metric	Original	PCA	gLPCA	NMF	GNMF	NTF	LRNTF	GLTD	NTD	GNTD	NTT	GNTT	TR	NTR	GNTR
ORL	AC	67.0	61.2	66.1	67.9	73.3	66.9	74.4	63.2	67.6	73.6	53.0	69.1	48.4	66.7	**75.8**
	NMI	83.2	78.9	80.2	82.9	86.8	81.2	87.2	78.8	82.2	86.3	72.5	83.7	67.6	82.0	**87.8**
FEI PART 1	AC	54.4	55.3	54.7	56.8	63.4	51.8	64.3	51.8	60.1	68.1	33.4	61.3	43.0	69.7	**73.4**
	NMI	75.6	75.0	72.6	75.9	80.5	73.5	81.8	61.5	77.4	83.5	55.4	78.2	65.0	84.4	**86.6**
GT	AC	44.9	41.6	41.7	45.5	44.0	42.1	47.5	40.9	43.5	40.3	42.1	40.3	30.1	47.4	**51.9**
	NMI	62.5	59.1	59.6	63.1	62.2	60.8	64.9	59.8	61.8	59.3	63.3	60.8	50.9	65.5	**68.5**
COIL-100 PART 1	AC	70.4	70.0	70.0	69.5	74.6	69.1	76.4	70.6	73.5	77.1	75.8	80.0	70.4	72.9	**84.2**
	NMI	80.2	79.4	79.6	80.2	85.0	78.8	84.6	80.2	79.3	86.2	83.7	88.8	78.7	81.5	**90.5**
Faces94 PART 1	AC	74.4	71.6	71.7	77.1	76.5	77.6	75.7	73.2	76.9	75.8	75.8	74.1	**80.8**	80.5	77.6
	NMI	90.9	88.6	88.4	91.3	92.1	91.5	91.9	89.7	91.2	92.2	89.7	91.5	92.6	**92.9**	**92.9**

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