Echo state network によるオーロラ電流非線型応答の解析 中野 慎也 モデリング研究系 准教授



The auroral electrojet is an electric current accompanied by auroral activity. Although it is known that the auroral electrojet is controlled by solar wind conditions, the relationship between the solar-wind state and the electrojet strength has not completely resolved. To investigate nonlinear effects of the solar wind, we modelled the relationship between the solar wind and the auroral electrojet with an echo state network model, which is a kind of recurrent neural network.

Input variables:

Echo state network

We employ an echo state network for approximating the relationship between the solar wind variables and the *AU* and *AL* index which is a geomagnetic index representing auroral activity (World Data Center for Geomagnetism, Kyoto et al., 2015).

Here we update each state variable $x_{k,i}$ at each time step as

$$x_{k,i} = \frac{1}{2} x_{k-1,i} + \frac{1}{2} \tanh(\boldsymbol{w}_i^T \boldsymbol{x}_{k-1} + \boldsymbol{u}_i^T \boldsymbol{z}_k + \boldsymbol{\eta}_i)$$

where the parameters w_i , u_i , and η_i were randomly determined in advance and fixed.

The output for the time step k is then obtained as follows:

$$y_k = \boldsymbol{\beta}^T \boldsymbol{x}_k.$$

We determine the weight β is determined so that the following objective function is minimized:

Solar-wind magnetic field (B_x , B_y , B_z), solar-wind speed (V_{sw}), density (N_{sw}), temperature (T_{sw})

Output variables:

Two geomagnetic indies (AU and AL) representing auroral activity

$$J = \sum_{k=1}^{K} || \boldsymbol{d}_{k} - \boldsymbol{y}_{k} ||^{2}.$$

The minimization is achieved with an ensemble-based variational method (e.g., Nakano, 2021). The model is explained in more detail in our recent paper (Nakano and Kataoka, 2022).



Analysis of nonlinear effects







Comparison of some ESN outputs during the period from 21 October to 25 October 1999. The top panel shows the ESN output with all the parameters (red), the synthetic indices where the solar-wind speed effect was turned off (green), those where the solar-wind density effect was turned off (blue), and the observed AU and AL indices (gray). The second panel shows the IMF Bx (green), By (blue), and Bz (red) in GSM coordinates. The third panel shows the solar wind speed, the fourth panel shows the solar wind density.

We define the effect of solar-wind density N as $\Delta AU_{Neff} = \left[AU(N = N_{obs} + 0.11) - AU(N = N_{obs})\right] / 0.1,$ $\Delta AL_{Neff} = \left[AL(N = N_{obs} + 0.11) - AL(N = N_{obs})\right] / 0.1.$

The left two panels are two-dimensional histograms indicating the density effect against the solar-wind speed. The right panels are histograms indicating the density effect against the north-south component of the solar-wind magnetic field.

References

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