2021年6月18日　統計数理研究所　オープンハウス

Residual Analysis for State-Space Models

Xun Shen　総合研究大学院大学　統計科学専攻　博士課程（5年一貫制）4年

Identifying the State Space Models with Nonlinear Response

$$x_{t+1} = Fx_t + Hu_t + v_t : v_t \sim N(0, Q) \quad \text{State equation}$$

$$y_t = C + Gx_t + Ju_t + s(x_t) + w_t : w_t \sim N(0, R) \quad \text{Observation equation}$$

$s(x_t)$: Unknown nonlinear response

Problem 1. Given the input set $U_T = \{u_0, \ldots, u_T\}$ and observation set $Y_T = \{y_0, \ldots, y_T\}$, to obtain $\hat{\theta}_m^*$ and $\hat{s}^*(x_t)$ which satisfies

$$\max \log p_{y_0}(y_0) + \sum_{t=0}^{T} \log p_{y_{t+1}}\left(y_{t+1} \middle| Y_t, U_t, \hat{\theta}_m, \hat{s}(x_t)\right).$$

Non-parametric model

$\hat{s}_{\mathrm{np}}(x_t) = \sum_{i=1}^{N_{\mathrm{np}}} \beta_i h(x_t, a_i, b_i)$, $\hat{\theta}_s$ is the vector of unknown parameters in β_i, a_i, b_i.

Problem 2. Given the input set $U_T = \{u_0, \ldots, u_T\}$ and observation set $Y_T = \{y_0, \ldots, y_T\}$, to obtain $\hat{\theta}_m^*$ and $\hat{\theta}_s^*$ which satisfies

$$\max \log p_{y_0}(y_0) + \sum_{t=0}^{T} \log p_{y_{t+1}}\left(y_{t+1} \middle| Y_t, U_t, \hat{\theta}_m, \hat{\theta}_s\right).$$

We propose a Expectation-Maximization (EM) algorithm-based method to identify the state space models with nonlinear response

1. **Kalman filter (Expectation).** Estimate the state with linear response and input.
2. **Reconstruction (Maximization I).** Identify the nonlinear response in the model using nonparametric methods such as kernel functions or neural network models.
3. **Parametrization (Maximization II).** Use the MLE method or EM algorithm to estimate the parameters in the linear part of the state space model

Application in Battery Capacity Estimation

$$U_{e,t+1} = e^{-\frac{\Delta\tau}{R_e C_e}} U_{e,t} + \left(1 - e^{-\frac{\Delta\tau}{R_e C_e}}\right) R_e i_t + v_{1,t}$$

$$S_{t+1} = S_t + \frac{\eta_c \Delta\tau}{C_n} i_t + v_{2,t}$$

$$U_t = U_{OCV,0} + K_0 S_t + s(S_t) + R_0 i_t + U_{e,t} + w_t$$