Crustal strain-rate fields estimated from GNSS data with a Bayesian approach and its correlation to seismic activity

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【Introduction】

We proposed a new Bayesian approach to estimate continuous crustal strain-rate fields from spatially discrete displacement-rate data, based on Global Navigation Satellite System (GNSS) observations, under the prior constraint on spatial flatness of the strain-rate fields. The optimal values of the hyperparameters in the model of strain-rate fields are determined by using Akaike's Bayesian Information Criterion. A methodological merit of this approach is that, by introducing a two-layer Delaunay tessellation technique, the time-consuming computation of strain rates can be omitted through the model estimation process. We applied the Bayesian approach to GNSS displacement-rate data in Mainland China to obtain a unique GNSS velocity field. We also examined the correlation between the estimated strain-rate fields and seismic activity by using Molchan's Error Diagram. The results show that the increase rate of maximum shear strain is positively correlated with the occurrence of earthquakes, indicating the strain rate can be used to augment probability earthquake models for background seismicity forecasting.

【Method】

Observation equation

- Observed velocity vector $U_0 = \left[v_{0x}^{(1)}, v_{0y}^{(1)}, \cdots, v_{0x}^{(N_s)}, v_{0y}^{(N_s)}\right]^{\mathrm{T}}$
- True velocity vector $U = \left[v_{x}^{(1)}, v_{y}^{(1)}, \cdots, v_{x}^{(N_s)}, v_{y}^{(N_s)}\right]^{\mathrm{T}}$
- Supposing that $U_0 - U \sim \mathrm{Normal}(0, W)$
- Distribution of observations U_0 conditioning on U

$$P(U_0|U) = [\det(2\pi W)]^{-1/2} \exp\left[-\frac{1}{2}(U_0 - U)^{\mathrm{T}} W^{-1} (U_0 - U)\right]$$

Transformation of velocity into strain rate

- strain rate tensor $E = RU$

$$E = \left[\dot{\varepsilon}_{xx}^{(1)}, \dot{\varepsilon}_{yy}^{(1)}, \dot{\varepsilon}_{xy}^{(1)}, \cdots, \dot{\varepsilon}_{xx}^{(N_{tr})}, \dot{\varepsilon}_{yy}^{(N_{tr})}, \dot{\varepsilon}_{xy}^{(N_{tr})}\right]^{T}$$

Smoothness prior

- Assuming $F = \mathbf{D}E$.
- F follows a normal distribution with zero mean

$$F \sim \mathrm{Normal}(0, \mathbf{W_2})$$
$$\mathbf{DR}U \sim \mathrm{Normal}(0, \mathbf{W_2})$$

- Prior pdf on the smoothness of U as

$$Q(U) = \left[\det{}^{+}\left(\frac{1}{2\pi}\mathbf{R}^{\mathrm{T}}\mathbf{D}^{\mathrm{T}}\mathbf{W_2}^{-1}\mathbf{DR}\right)\right]^{\frac{1}{2}} \exp\left[-\frac{1}{2}U^{\mathrm{T}}\left(\mathbf{R}^{\mathrm{T}}\mathbf{D}^{\mathrm{T}}\mathbf{W_2}^{-1}\mathbf{DR}\right)U\right]$$

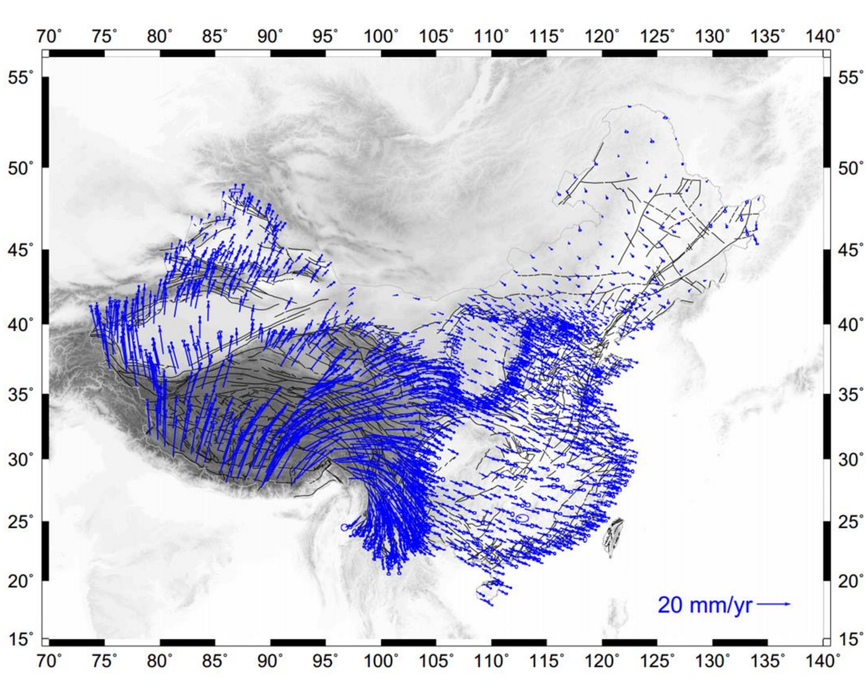
Posterior distribution

- Posterior pdf of U $\quad P(U|U_0) = \left(P(U_0|U)Q(U)\right)/\left(P(U_0)\right)$
- Optimal estimate of displacement vector

$$\hat{U} = \left(\mathbf{R}^{\mathrm{T}}\mathbf{D}^{\mathrm{T}}\mathbf{W_2}^{-1}\mathbf{DR} + \mathbf{W}^{-1}\right)^{-1}\mathbf{W}^{-1}U_0$$

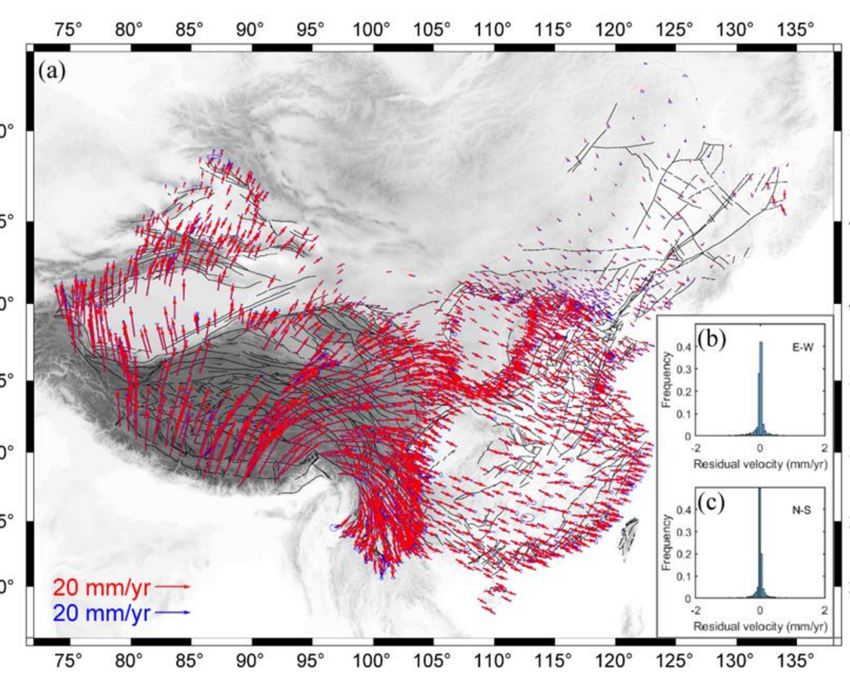
- Optimal estimate of the strain rate tensor $\hat{E} = \mathbf{R}\hat{U}$

【Data】



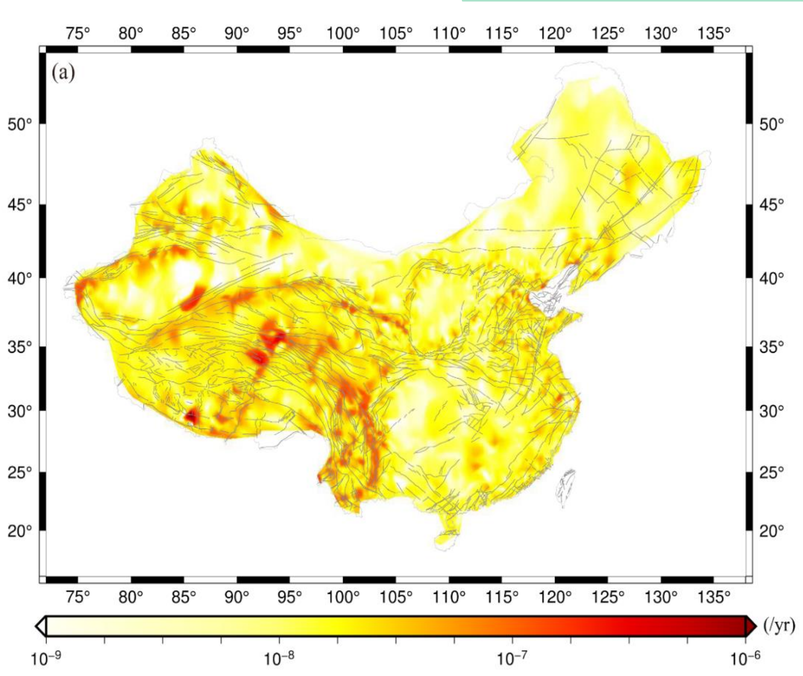
- GNSS velocity field of Mainland China from 1999 to 2017.
  - The blue arrows indicate GNSS velocity vectors at each station. The gray lines represent the traces of faults

【Results】

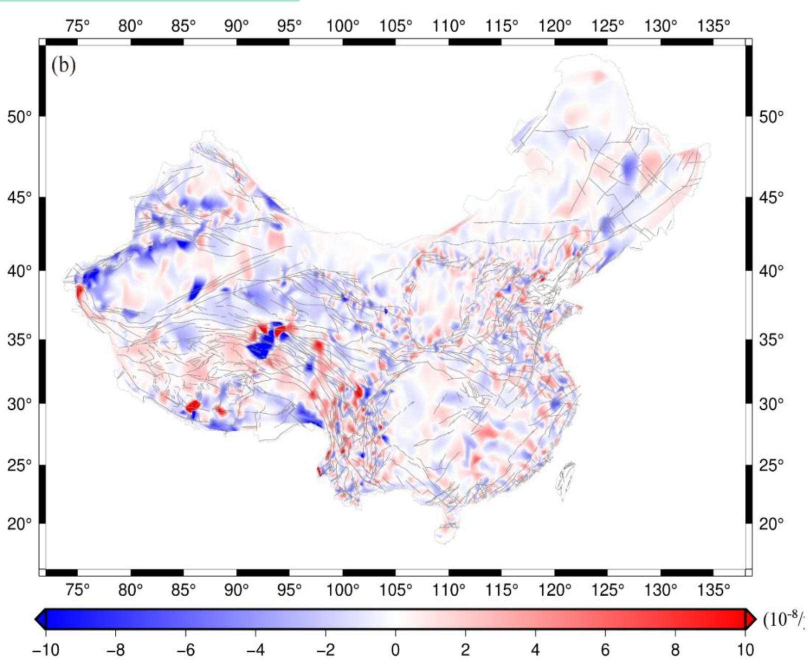


- (a) The differences between observed GNSS velocities and estimated velocities in Mainland China.
- The insets (b) and (c) are the histograms of differences between the observed and estimated velocities for the longitudinal and latitudinal components, respectively.

Strain rates in Mainland China

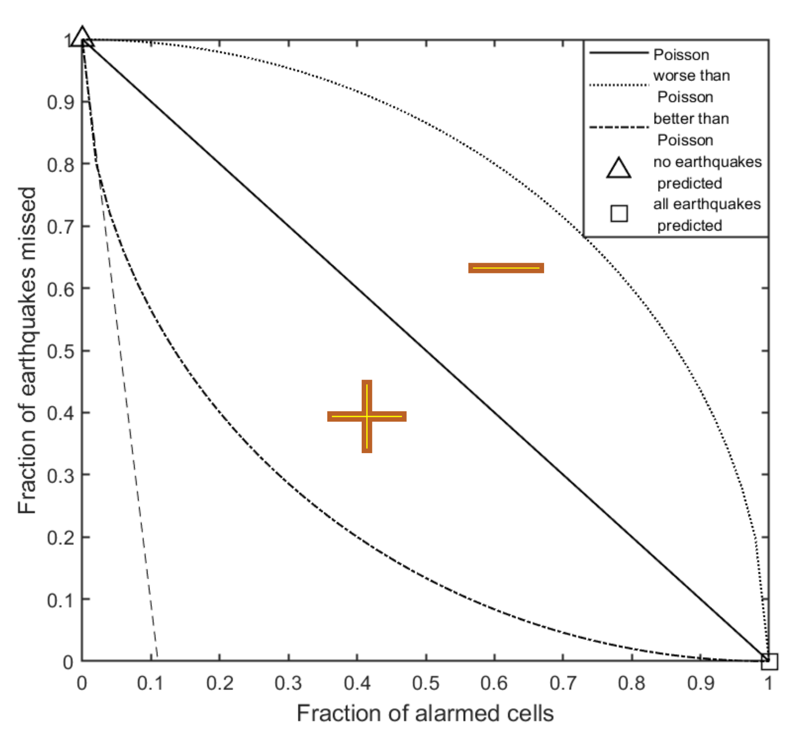


maximum shear strain rates

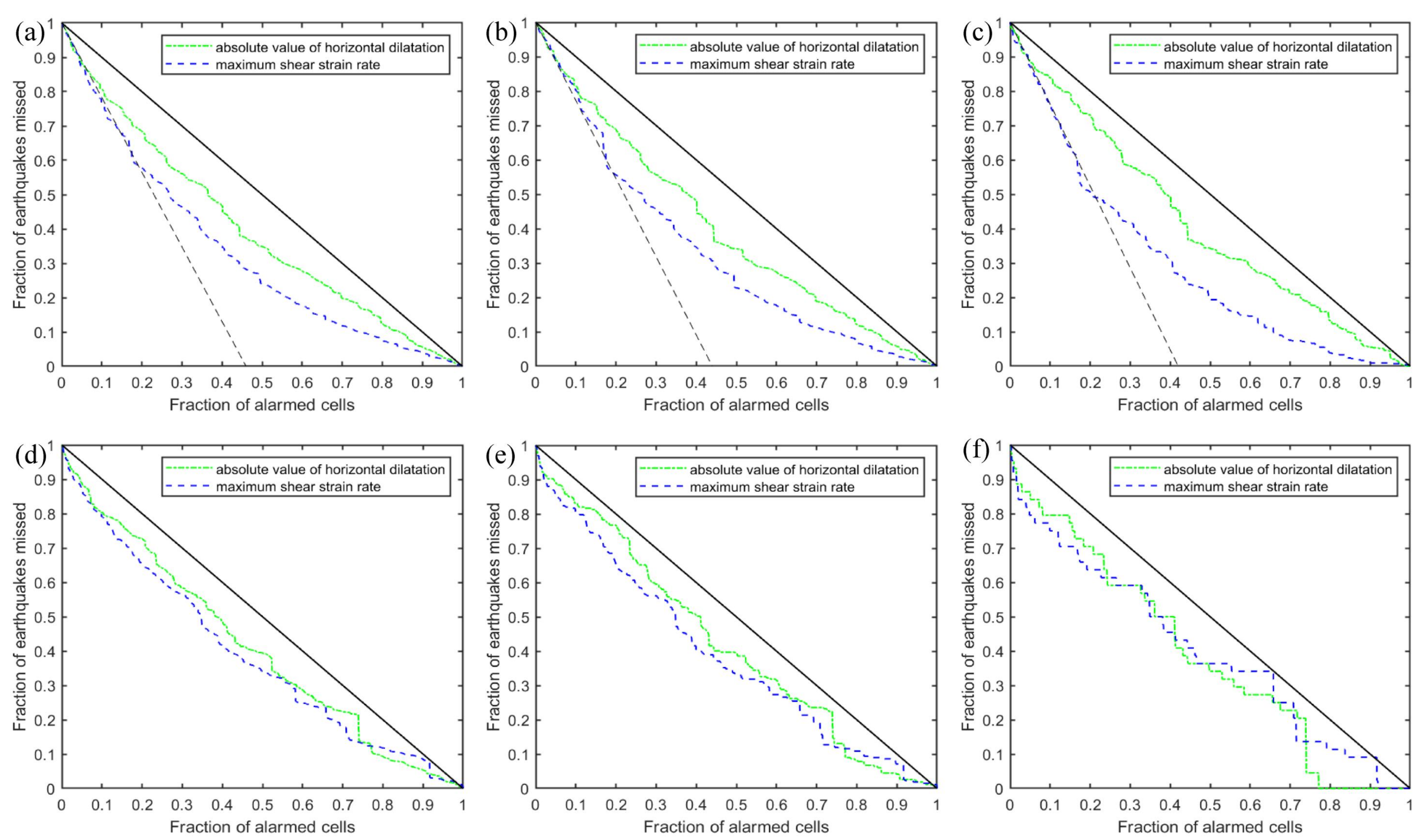


horizontal dilation rates

【Correlation between GNSS strain rate and seismicity rate】



- A demonstration of the Molchan error diagram. The dashed straight line represents the tangent line of the curve.
  - If the curve below the diagonal straight line, the model is better than completely random guess. The larger the slope, the better the performance of the earthquake prediction.



- **Molchan error diagrams** for verifying the correction between strain-rate and seismicity.
  - Diagrams (a) to (c) show the correlation between strain rates and seismic events in the earthquake catalog from January 1, 1999 to January 1, 2017.
  - Diagrams (d) to (f) show the correlation between strain rates and seismic events in the earthquake catalog from January 1, 2017 to November 30, 2019.

【Conclusion】

In this Bayesian approach, the second-order smoothness prior is calculated by the interpolation of the velocity data through a two-layered Delaunay tessellation. Reducing the model estimation to an algebraic problem, we can make the algorithm computationally efficient.

Through the Molchan error diagram analysis, the positive correlation between the estimated strain rate and earthquake occurrence was verified. In particular, the maximum shear strain rate is positively correlated with the seismicity. These results show that the maximum shear strain rate is an important factor in earthquake prediction and seismic activity research.