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資産取引ゲームにおける必勝戦略

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【概要】
資産取引ゲームにおける成行取引 (market order), 指値取引 (limit order) それぞれについて，資産価格系列に対して測度論的な仮定は一切せずに直接判定できる条件 (成行取引では weak stationarity, 指値取引では strong jaggedness) のもとで，取引を繰り返すにつれて幾らでも所持資金が増大する具体的な取引戦略を示し，株式市場の実際のデータによって提示した取引戦略の有効性を実証する．これらを竹内　啓先生との共同研究の経過をまとめた以下の論文から紹介する．

K. Takeuchi and M. Kumon
Winning strategies for asset trading games.
ISM Research Memorandum, No. 1213, 2021.

【単一資産取引ゲームの成行取引必勝戦略】

$S(t) > 0$: price of the unit amount of the asset $\mathcal{S}$ at time $t \geq 0$

$0 \leq t_0 < t_1 < t_2 < \cdots$: Investor's market order discrete time points

$\{S_n = S(t_n)\};\ n = 0, 1, 2, \ldots$: sequence of prices at the end of the n-th trading period of Investor

K_n : Investor's capital at the end of the n-th period with the initial capital K_0

$\Rightarrow\ K_n = K_{n-1}(1 + \alpha_n(X_n - 1));\ 1 \geq \alpha_n \geq 0,\ X_n = \frac{S_n}{S_{n-1}} > 0$

$\{\alpha_n = \alpha_n(S_0, \ldots, S_{n-1})\};\ n = 1, 2, \ldots$: Investor's trading strategy

□ Weak stationarity of $\{S_n\};\ n = 0, 1, 2, \ldots$

1. Boundedness : $0 < \exists c < \exists d < \infty$ s.t. $c \leq S_n \leq d\ \ \forall n = 0, 1, 2, \ldots$
2. Variability : $\{S_n\}$ reasonably varies in the sense $D_n = \sum_{i=1}^n (X_i - 1)^2 \to \infty$ as $n \to \infty$

$\Rightarrow\ \exists\{\alpha_n\}$ s.t. $K_n \to \infty$ as $n \to \infty$

The constant $\alpha = 1/2$ strategy : nearly optimal with exponential growth rate of K_n (rebalanced portfolio)

【複数資産取引ゲームの成行取引必勝戦略】

$\{S^a(t) > 0\};\ a = 1, \ldots, m$: prices of the unit amount of the m assets $\{\mathcal{S}^a\}$ at time $t \geq 0$

$\{S_n^a = S^a(t_n)\};\ a = 1, \ldots, m;\ n = 0, 1, 2, \ldots$: sequences of prices of the unit amount of the m assets $\{\mathcal{S}^a\}$ at the end of the n-th trading period of Investor

$\Rightarrow\ K_n = K_{n-1}\left(1 + \sum_{a=1}^m \alpha_n^a (X_n^a - 1)\right);\ \alpha_n^a \geq 0,\ \sum_{a=1}^m \alpha_n^a \leq 1,$
$X_n^a = \frac{S_n^a}{S_{n-1}^a} > 0;\ a = 1, \ldots, m$

$\{\alpha_n^a = \alpha_n^a(\{S_0^b\}, \ldots, \{S_{n-1}^b\})\};\ a, b = 1, \ldots, m;\ n = 1, 2, \ldots$: Investor's trading strategy

□ Weak stationarity of $\{S_n^a\};\ a = 1, \ldots, m;\ n = 0, 1, 2, \ldots$

1. Boundedness : $0 < \exists c < \exists d < \infty$ s.t. $c \leq S_n^a \leq d\ \ \forall a = 1, \ldots, m;\ \forall n = 0, 1, 2, \ldots$
2. Variability : $\{S_n^a\}$ reasonably varies in the sense $\sum_{i=1}^n (X_i^a - 1)^2 \to \infty,\ \sum_{i=1}^n (X_i^a - 1)^2 \sum_{i=1}^n (X_i^b - 1)^2 (1 - r_{nab}^2) \to \infty$ as $n \to \infty$ $\quad r_{nab} = \frac{\sum_{i=1}^n (X_i^a - 1)(X_i^b - 1)}{(\sum_{i=1}^n (X_i^a - 1)^2 \sum_{i=1}^n (X_i^b - 1)^2)^{\frac{1}{2}}};\ a \neq b = 1, \ldots, m$
i.e. squares of correlation coefficients $r_{nab}^2 < 1;\ a \neq b = 1, \ldots, m$

$\Rightarrow\ \exists$many $\{\alpha_n^a\}$ s.t. $K_n \to \infty$ as $n \to \infty$

e.g. constant $(1/m, \ldots, 1/m)$ strategy etc.

【単一資産取引ゲームの高頻度指値取引必勝戦略】

$0 \leq t_0 < t_1 < t_2 < \cdots$: Limit order discrete time points recursively after t_{i-1} is determined, Investor trades the asset at the first time either $\frac{S(t_i)}{S(t_{i-1})} = 1 + \delta = r$ or $\frac{S(t_i)}{S(t_{i-1})} = \frac{1}{1+\delta} = \frac{1}{r};\ \delta > 0,\ r > 1;\ i = 1, 2, \ldots$

$\Rightarrow$ asset trading game reduces to coin tossing game

$$\xi_i = \begin{cases} 1 & \text{if } X_i = r \\ -1 & \text{if } X_i = r^{-1} \end{cases} \qquad \alpha_i^{\pm} = \begin{cases} \alpha_i^+ = \frac{\hat{p}_i^+(r+1)-1}{r-1} & \text{if } \xi_{i-1} = 1 \\ \alpha_i^- = \frac{\hat{p}_i^-(r+1)-1}{r-1} & \text{if } \xi_{i-1} = -1 \end{cases}$$

$\hat{p}_i^+ = \frac{N_{i-1}^{++}+c_1}{N_{i-1}^{++}+N_{i-1}^{+-}+c_1+c_2},\quad \hat{p}_i^- = \frac{N_{i-1}^{-+}+c_1}{N_{i-1}^{-+}+N_{i-1}^{--}+c_1+c_2},\quad c_1, c_2 > 0,$

$N_{i-1}^{++},\ N_{i-1}^{-+},\ N_{i-1}^{+-},\ N_{i-1}^{--}$: numbers of pairs $(\xi_{j-1}\xi_j) = (1\ 1), (-1\ 1), (1\ -1), (-1\ -1);\ j = 2, \ldots, i-1$

$\{\alpha_i^{\pm}\}$: Markov strategy in coin tossing game

$\eta_k = \log(1 + \delta_k) = a^{-k};\ a > 1,\ k = 1, 2, \ldots$

$k \to \infty$: high-frequency limit order strategy

$d \log S(t) = \log S(t+dt) - \log S(t) = O(dt^H),\ H \in (0, 1]$: Hölder exponent

$D(p\|q) = p \log \frac{p}{q} + (1-p) \log \frac{1-p}{1-q}$: Kullback Leibler divergence

When Investor takes the high-frequency Markov limit order strategy

$$\frac{1}{n_k} \log \frac{K_{n_k}(\alpha_i^{\pm})}{K_0} \to D\left(\frac{1}{2^{1/H-1}} \middle\| \frac{1}{2}\right) \text{ as } k \to \infty$$

$H \neq 1/2$ ($H < 1/2$: strong jaggedness) $\Rightarrow$ $K_{n_k} \to \infty$ with exponential growth rate $D\left(\frac{1}{2^{1/H-1}} \middle\| \frac{1}{2}\right)$ as $k \to \infty$

【株式データの数値例】

□ S1:Takeda S2:Toyota S3:NSC S4:Kirin S5:Kajima S6:Yokohama S7:Mitsubishi S8:Canon の日次株価終値に対する一定比率成行取引戦略

図1:各 D_n 図2:各 K_n 図3:各複数資産 K_n 図4:(S5,S8,S6) の各相関係数

□ Sony, IHI の分単位株価に対する高頻度マルコフ指値取引戦略

図5:Sony の Hölder 指数 図6:Sony に対して $K_n \geq 10^3 K_0$ (2170 minutes)

図7:IHI の Hölder 指数 図8:IHI に対して $K_n \geq 10^3 K_0$ (5229 minutes)

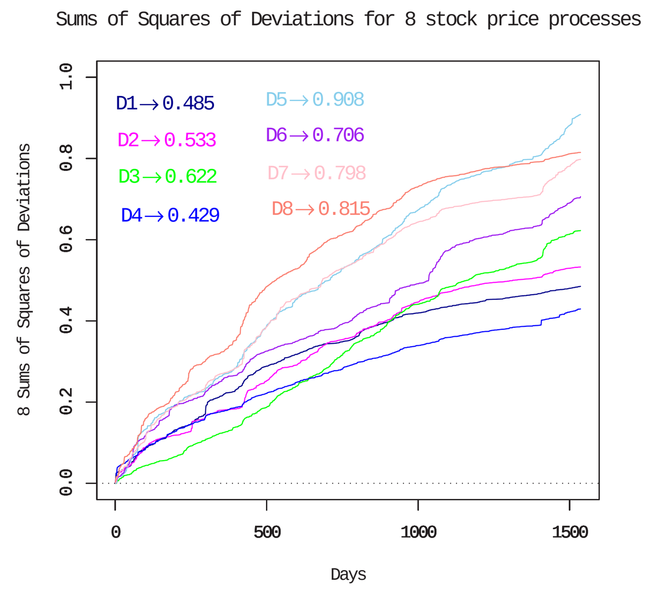

図 1: **Sums of squares of deviations**

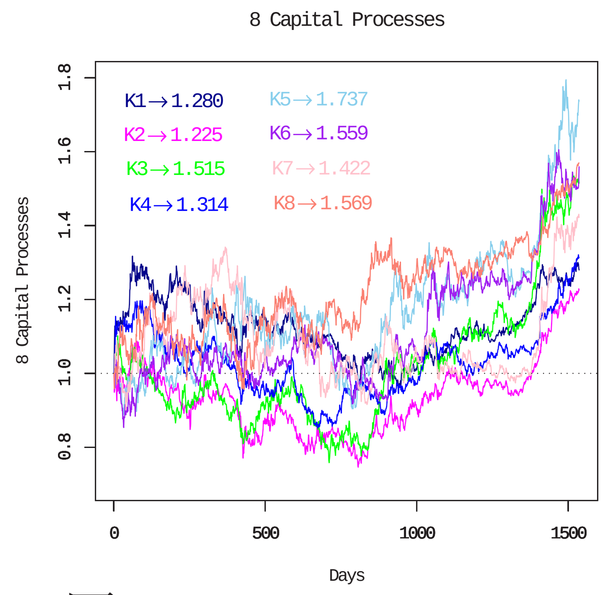

図 2: **Capital processes**

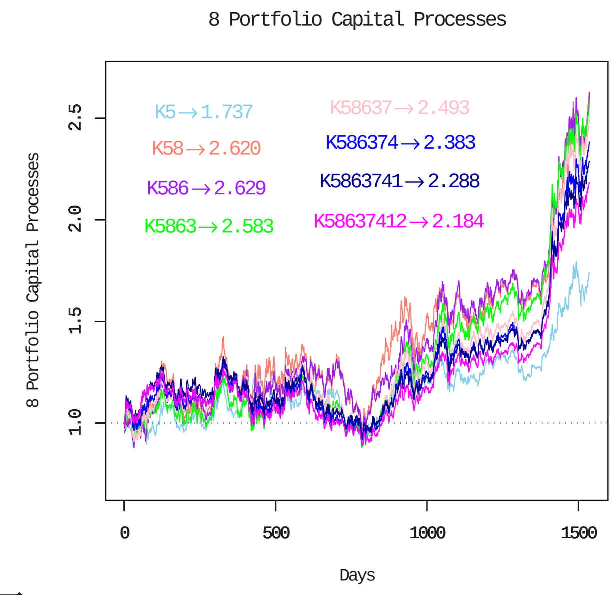

図 3: **Portfolio capital precesses**

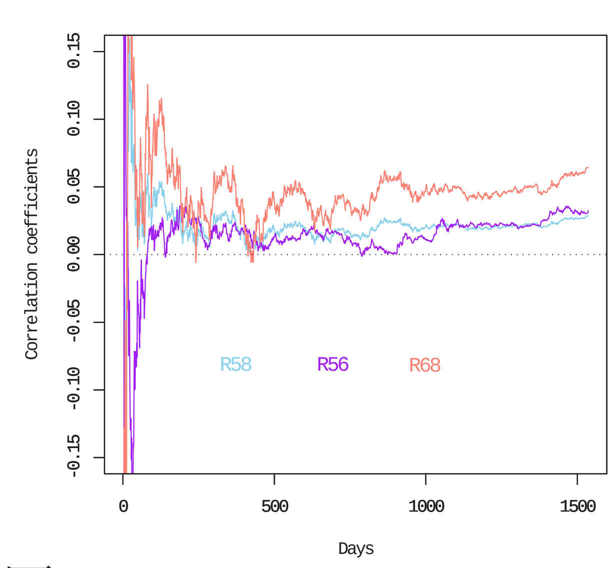

図 4: **Correlation coefficients**

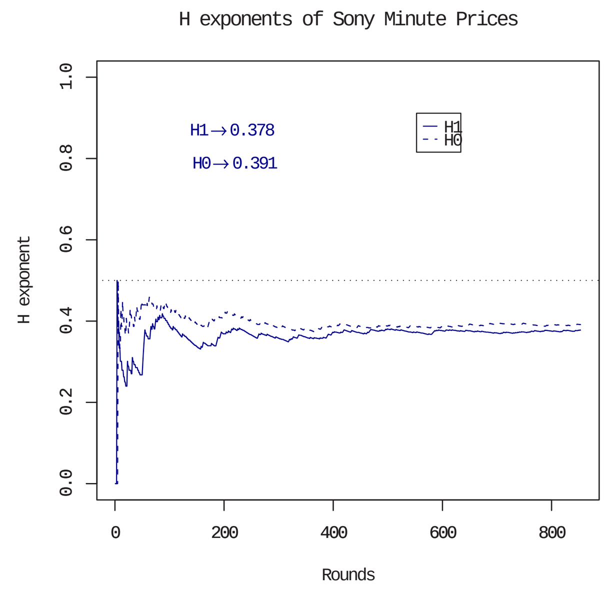

図 5: **Hölder exponents of Sony**

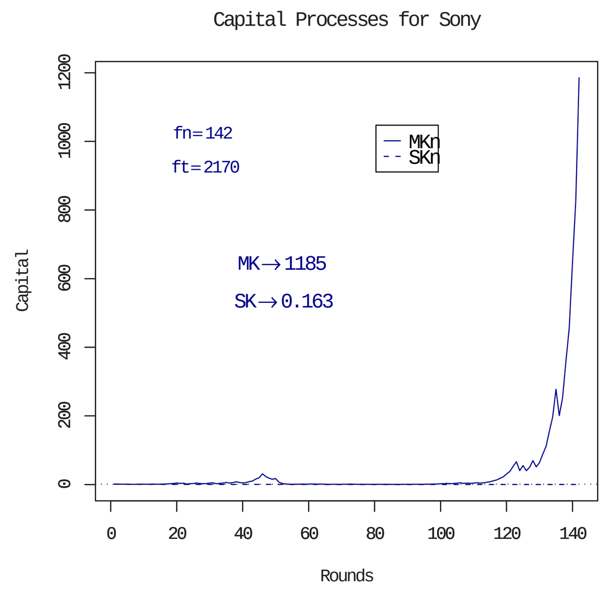

図 6: **Capital processes for Sony**

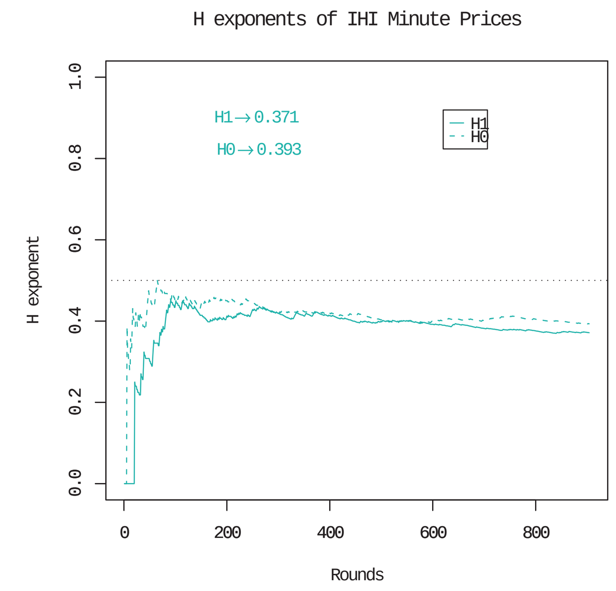

図 7: **Hölder exponents of IHI**

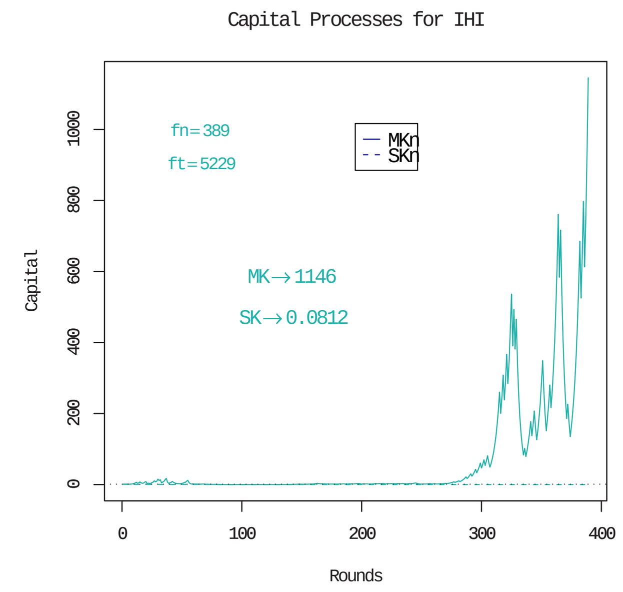

図 8: **Capital processes for IHI**