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空間的非線形評価を伴う受粉サービスに対する離散最適化

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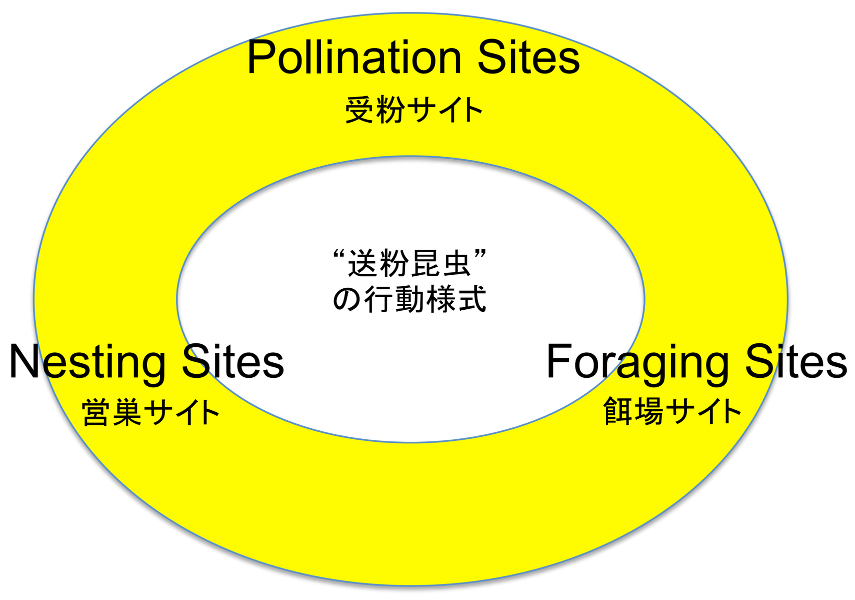
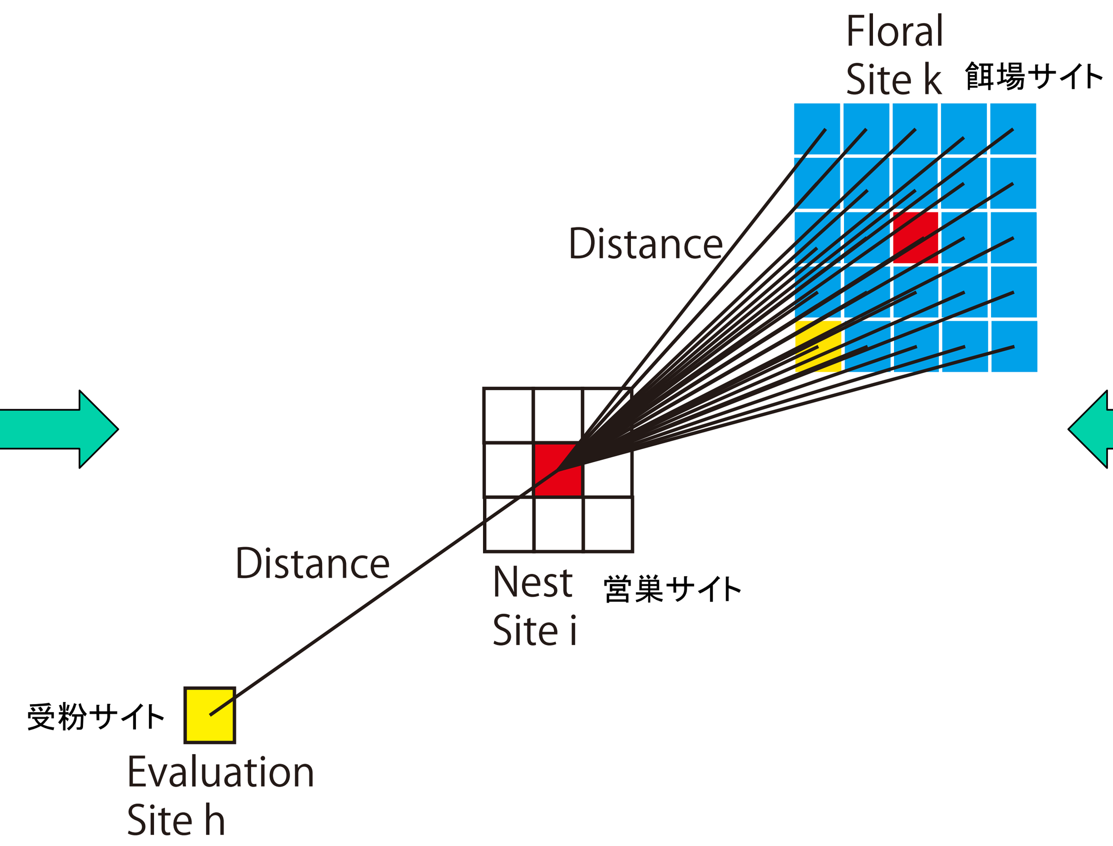
【受粉サービス】

受粉を必要とする農作物の生産では、ミツバチなど"送粉昆虫"の果たす役割は生態系がもたらす重要な"生態系サービス"の一つである。ミツバチが蜜・花粉を求めて飛び回る際、体毛に花粉が付着し、雌しべの先につくことにより受粉が完了する。また、閉鎖環境のハウスなどにおいても、野生の送粉昆虫による受粉ではないものの、花の開花時期に合わせて花粉交配専用の飼養ミツバチを入手し、ハウス内に巣箱を設置することによりミツバチの飛翔を促し、受粉が完了する。





(http://www.kutikshoney.com/crop_pollination.htm)


On Cucumber Blossom


On Apple Blossom

【研究目的】

本研究では、土地利用の改変に伴う効率的な花粉交配を念頭に蜂場・蜜源の最適配置を探索できる離散最適化システムの構築を目指し、限られた送粉昆虫資源に対し受粉サービスの効率的・効果的な供給を可能にする土地利用管理を探求する。

【 受粉スコア(Pollination Score by Landsdorf et al., Ann. Bot., 2009)】

"Since pollinator abundance is limited by both nesting and floral resources, the pollinator score on a parcel is simply ***the product of foraging and nesting***. This score represents the location and relative abundance of pollinators available for crop pollination from a parcel."

= Habitat Suitability for Nesting × Habitat Suitability for Foraging

Habitat suitability for Nesting by a pollinator at *k*-parcel

$$HN_k = \sum_{j=1}^{J} N_j p_{jk} = \sum_{j=1}^{J} \nu_k(j)$$

j: j-habitat (land use) J: the number of habitats for a parcel

k: k-parcel

N_j: Compatibility of j-habitat for a pollinator's nesting

p_{jk}: Proportion by j-habitat at k-parcel

×

Habitat suitability for Foraging for a pollinator at *k*-parcel

$$HF_k = \frac{\sum_{m=1}^{M}\sum_{j=1}^{J} F_j p_{jm} \exp(-D_{mk}/\alpha)}{\sum_{m=1}^{M} \exp(-D_{mk}/\alpha)} = \frac{\sum_{m=1}^{M}\sum_{j=1}^{J} \varphi_m(j) \exp(-D_{mk}/\alpha)}{\sum_{m=1}^{M} \exp(-D_{mk}/\alpha)}$$

foraging frequency in a parcel declines exponentially with distance

D_{mk}: Distance from k-parcel to m-parcel

F_j: Compatibility of j-habitat for a pollinator

α: Expected pollinator foraging distance

M: The number of parcels

Pollination Score induced by a pollinator at k-parcel ⟹ $P_k = HN_k \cdot HF_k$

Pollination Service at h-parcel by a pollinator ⟹ $PS_h = \{\sum_{k=1}^{M} P_k \exp(-D_{hk}/\alpha)\} / \sum_{k=1}^{M} \exp(-D_{hk}/\alpha)$

Decision variables: Land use selection for a parcel

$$x_{ij} = \begin{cases} 1 & \text{if the } j\text{-th habitat is selected for the } i\text{-th parcll} \\ 0 & \text{otherwise} \end{cases}, \quad \sum_{j=1}^{J} x_{ij} = 1 \ \forall i$$

Pollination Score expressed by decision variables

$$P_k = \sum_{m=1}^{M}\sum_{l=1}^{J}\sum_{j=1}^{J} \nu_l x_{kl} \varphi_j x_{mj} \exp(-D_{mk}/\alpha) / \sum_{m=1}^{M} \exp(-D_{mk}/\alpha)$$

$\nu_k(l) = \nu_l x_{kl}$: Nesting score for l-habitat at k-parcel

$\varphi_m(j) = \varphi_j x_{mj}$: Foraging score for j-habitat at m-parcel

Nonlinearity to linearity by a new variable

$$\boldsymbol{Y} = \boldsymbol{X} \otimes \boldsymbol{X}, \quad y_{M(i-1)+k, J(j-1)+l} = x_{ij} \cdot x_{kl} = x_{kl} \cdot x_{ij} = y_{M(k-1)+i, J(l-1)+j}$$

Constraints among variables from habitat nest

If $x_{ij} = 0 \Rightarrow x_{ij}\boldsymbol{X} = \boldsymbol{0}_m$. $\quad \sum_{l=1}^{J} y_{M(i-1)+k, J(j-1)+l} = 0 \ (k = 1, 2, \cdots, M)$

If $x_{ij} = 1 \Rightarrow x_{ij}\boldsymbol{X} = \boldsymbol{1}_m$. $\quad \sum_{l=1}^{J} y_{M(i-1)+k, J(j-1)+l} = 1 \ (k = 1, 2, \cdots, M)$

$$\begin{pmatrix} x_{11}\boldsymbol{X} & \cdots & x_{1J}\boldsymbol{X} \\ \vdots & \ddots & \vdots \\ x_{M1}\boldsymbol{X} & \cdots & x_{MJ}\boldsymbol{X} \end{pmatrix}$$

$$\sum_{l=1}^{J} y_{M(i-1)+k, J(j-1)+l} = x_{ij} \quad (i = 1, 2, \cdots, M,\ j = 1, 2, \cdots, J, k = 1, 2, \cdots, M)$$

Constraints among variables from habitat floral site

If $x_{kl} = 0 \Rightarrow \boldsymbol{X}x_{kl} = \boldsymbol{0}_M$. $\quad \sum_{j=1}^{J} y_{M(k-1)+i, J(l-1)+j} = 0 \ (i = 1, 2, \cdots, M)$

If $x_{kl} = 1 \Rightarrow \boldsymbol{X}x_{kl} = \boldsymbol{1}_M$. $\quad \sum_{j=1}^{J} y_{M(k-1)+i, J(l-1)+j} = 1 \ (i = 1, 2, \cdots, M)$

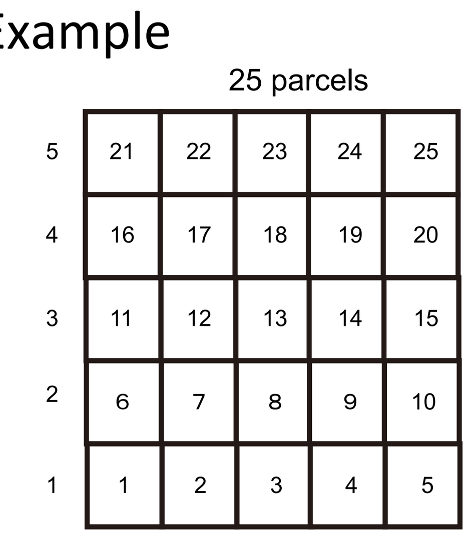
$$\begin{pmatrix} x_{11}\boldsymbol{X} & \cdots & x_{1J}\boldsymbol{X} \\ \vdots & \ddots & \vdots \\ x_{M1}\boldsymbol{X} & \cdots & x_{MJ}\boldsymbol{X} \end{pmatrix}$$

$$\sum_{j=1}^{J} y_{M(k-1)+i, J(l-1)+j} = x_{k,l} \quad (i = 1, 2, \cdots, M,\ l = 1, 2, \cdots, J, k = 1, 2, \cdots, M)$$

【モデリングと結果】

Example

	ν_i	φ_i
1 Forest	1	1
2 Coffee	0.1	0.5
3 Cane	0	0
4 Pasture/grass	0.1	0.2
5 Scrub	0.2	0.3
6 Bare	0.1	0.1
7 Built	0.1	0.3



1. Maximize pollination service only

$$J = \max \sum_{h=1}^{M} PS_h = \sum_{h=1}^{M}\sum_{i=1}^{M}\sum_{k=1}^{M} a_{ik}^h \boldsymbol{1}'_{J^2} \boldsymbol{z}_{M(i-1)+k}$$

subject to

$$\boldsymbol{Z} = \boldsymbol{Y}\{\boldsymbol{I}_n \otimes \mathrm{diag}(\varphi_1, \varphi_2, \cdots, \varphi_J)\}\{\mathrm{diag}(\nu_1, \nu_2, \cdots, \nu_J) \otimes \boldsymbol{I}_J\}$$

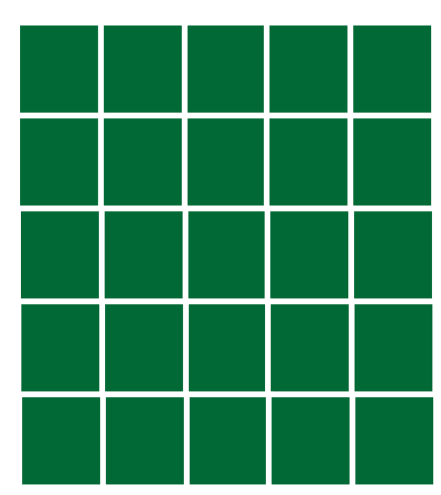
$x_{ij}\boldsymbol{X}\boldsymbol{1}_J = x_{i,j}\boldsymbol{1}_M \quad (i = 1, 2, \cdots, M,\ j = 1, 2, \cdots, J)$

$(\boldsymbol{X}x_{kl})\boldsymbol{1}_J = x_{k,l}\boldsymbol{1}_M \quad (k = 1, 2, \cdots, M, l = 1, 2, \cdots, J)$

$\sum_j x_{ij} = 1 \ \forall i$

where

$$a_{ik}^h = \frac{\exp\{-(D_{hi} + D_{ik})/\alpha\}}{\left(\sum_{i=1}^{m} \exp(-D_{hi}/\alpha)\right)\left(\sum_{l=1}^{m} \exp(-D_{il}/\alpha)\right)}$$



2.Maximize pollination service subject to limit for pollination service

$$J = \max \sum_{h=1}^{M} PS_h = \sum_{h=1}^{M}\sum_{i=1}^{M}\sum_{k=1}^{M} a_{ik}^h \boldsymbol{1}'_{J^2} \boldsymbol{z}_{M(i-1)+k}$$

subject to

$$\sum_{h=1}^{M}\sum_{i=1}^{M}\sum_{k=1}^{M} a_{ik}^h \boldsymbol{1}'_{J^2} \boldsymbol{z}_{M(i-1)+k} \le 5$$

$$\boldsymbol{Z} = \boldsymbol{Y}\{\boldsymbol{I}_J \otimes \mathrm{diag}(\varphi_1, \varphi_2, \cdots, \varphi_J)\}\{\mathrm{diag}(\nu_1, \nu_2, \cdots, \nu_J) \otimes \boldsymbol{I}_J\}$$

$x_{ij}\boldsymbol{X}\boldsymbol{1}_J = x_{ij}\boldsymbol{1}_M \quad (i = 1, 2, \cdots, M,\ j = 1, 2, \cdots, J)$

$(\boldsymbol{X}x_{kl})\boldsymbol{1}_J = x_{kl}\boldsymbol{1}_M \quad (k = 1, 2, \cdots, M, l = 1, 2, \cdots, J)$

$\sum_j x_{ij} = 1, \quad \forall i$

where

$$a_{ik}^h = \frac{\exp\{-(D_{hi} + D_{ik})/\alpha\}}{\left(\sum_{i=1}^{M} \exp(-D_{hi}/\alpha)\right)\left(\sum_{l=1}^{M} \exp(-D_{il}/\alpha)\right)}$$

3.Maximize benefits from land use subject to limit for pollination service

$$J = \max TB = \sum_{i=1}^{M}\sum_{j=1}^{J} b_j x_{ij}$$

subject to

$$\sum_{h=1}^{M}\sum_{i=1}^{M}\sum_{k=1}^{M} a_{ik}^h \boldsymbol{1}'_{J^2} \boldsymbol{z}_{M(i-1)+k} \ge 5$$

$$\boldsymbol{Z} = \boldsymbol{Y}\{\boldsymbol{I}_J \otimes \mathrm{diag}(\varphi_1, \varphi_2, \cdots, \varphi_J)\}\{\mathrm{diag}(\nu_1, \nu_2, \cdots, \nu_J) \otimes \boldsymbol{I}_J\}$$

$x_{ij}\boldsymbol{X}\boldsymbol{1}_J = x_{ij}\boldsymbol{1}_M \quad (i = 1, 2, \cdots, M,\ j = 1, 2, \cdots, J)$

$(\boldsymbol{X}x_{kl})\boldsymbol{1}_J = x_{kl}\boldsymbol{1}_M \quad (k = 1, 2, \cdots, M, l = 1, 2, \cdots, J)$

$\sum_j x_{ij} = 1 \quad \forall i$

where

$$a_{ik}^h = \frac{\exp\{-(D_{hi} + D_{ik})/\alpha\}}{\left(\sum_{i=1}^{M} \exp(-D_{hi}/\alpha)\right)\left(\sum_{l=1}^{M} \exp(-D_{il}/\alpha)\right)}$$

	b_j
1 Forest	0.3
2 Coffee	1
3 Cane	1.2
4 Pasture/grass	0.7
5 Scrub	0.1
6 Bare	0
7 Built	0