Semi-parametric estimates of the long-term background trend, periodicity, and clustering effect in crime data

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【Abstract】

Past studies have shown that crime behaviors are clustered. This study proposes a spatiotemporal Hawkes-type point-process model, which includes a background component with daily and weekly periodization and a clustering component that is triggered by previous events, for describing the occurrences of violence or robbery related to crimes in the city of Castellon, Spain, during 2012 and 2013. A nonparametric method, called stochastic reconstruction, is used to estimate each component, including daily and weekly periodicity of background rate, spatial background rate, long-term background trend, and the spatial and temporal response function in the triggering component, of the conditional intensity of the model.

【Data】

This study investigates the data of violence or robbery related to crimes in the city of Castellon, Spain, during 2012 and 2013 (Figure 1).

【Model formulation】

We use a space-time point-process model to describe the data, which is completely specified by a conditional intensity function

$$\lambda(t,x,y) = \mu_0\, \mu_t(t)\, \mu_d(t)\, \mu_w(t)\, \mu_b(x,y) + A\int_{\infty}^{t}\iint_{R^2} g(t-s)\, f(x-u,y-v)\, N(ds\times du\times dv)\,,$$

where where t (day) and (x,y) (km) denote time and location, respectively, $\mu_t(t)$, $\mu_d(t)$, and $\mu_w(t)$ represent the trend term, the daily periodicity, the weekly periodicity in the temporal components of the background rate, respectively, $\mu_b(x,y)$ represents the spatial homogeneity of the background rate, μ_0 and A are constants, and $g(t-s)f(x-u,y-v)$ represents the intensity of the subprocess triggered by an event previously occurring at location (u,v) and time s. In the above, the average values of μ_t, μ_d, μ_w, and μ_b are all normalized to 1, g and f are normalized as probability densities.

【Estimation method and algorithm: Stochastic reconstruction】

We estimate μ_t, μ_d, μ_w, μ_b, g and f non-parametrically by using the stochastic reconstruction method proposed in [1,2,3]. Given a realization of the point process, $\{(t_i,x_i,y_i):\ i=1,2,\cdots,n\}$, These functions in the background component can be reconstructed in the following way:

$$\hat{\mu}_t(t) \propto \sum_i w_i^{(t)}\, I(t_i\in[t-\Delta,t+\Delta])\,, \qquad w_i^{(t)} = \frac{\mu_t(t_i)\,\mu_b(x_i,y_i)}{\lambda(t_i,x_i,y_i)}$$

$$\hat{\mu}_d(t) \propto \sum_i w_i^{(d)}\, I(t_i-\lfloor t_i\rfloor\in[t-\Delta,t+\Delta])\,, \qquad w_i^{(d)} = \frac{\mu_d(t_i)\,\mu_b(x_i,y_i)}{\lambda(t_i,x_i,y_i)}$$

$$\hat{\mu}_w(t) \propto \sum_i w_i^{(w)}\, I\left(t_i - 14\times\left\lfloor\frac{t_i}{14}\right\rfloor\in[t-\Delta,t+\Delta]\right), \qquad w_i^{(d)} = \frac{\mu_w(t_i)\,\mu_b(x_i,y_i)}{\lambda(t_i,x_i,y_i)}$$

$$\hat{\mu}_b(x,y) \propto \sum_i \varphi_i\ \ Z_h(x-x_i,y-y_i)\,, \qquad \varphi_i = \frac{\mu_0\,\mu_t(t_i)\mu_d(t_i)\mu_w(t_i)\,\mu_b(x_i,y_i)}{\lambda(t_i,x_i,y_i)}$$

$$\hat{g}(t) \propto \sum_{ij}\rho_{ij}\, I(t_j-t_i\in[t-\Delta,t+\Delta])\,, \qquad \rho_{ij} = \frac{Ag(t_j-t_i)h(x_j-x_i,y_j-y_i)}{\lambda(t_j,x_j,y_j)}, \qquad \text{for } j<i,$$

$$\hat{f}(x,y) \propto \sum_{ij}\rho_{ij}\, I(x_j-x_i\in[x-\Delta_x,x+\Delta_x])\,,$$

Once the above functions are estimated, we can update μ and A through maximizing the likelihood function

$$\log L = \sum_{i=1}^{n}\log\lambda(t_i,x_i,y_i) - \int_0^T\iint_S \lambda(s,u,v)\,du\,dv\,ds$$

An iterative algorithm is designed to estimate both the functions and the relaxation parameters, μ and A, simultaneously.

【Results】

The results show that the background rate of the occurrence process of violence or robbery related to crimes in the city of Castellon, Spain, during 2012 and 2013, includes clear daily and weekly periodicity (Figures 2 and 3). The reconstructed spatial and temporal response functions in the clustering component imply that, once a crime occurs, it likely trigger another crime within the coming 3 days and within 100 meters in distance. The parameter estimates are $\mu=0.771$ and $A=0.029$.

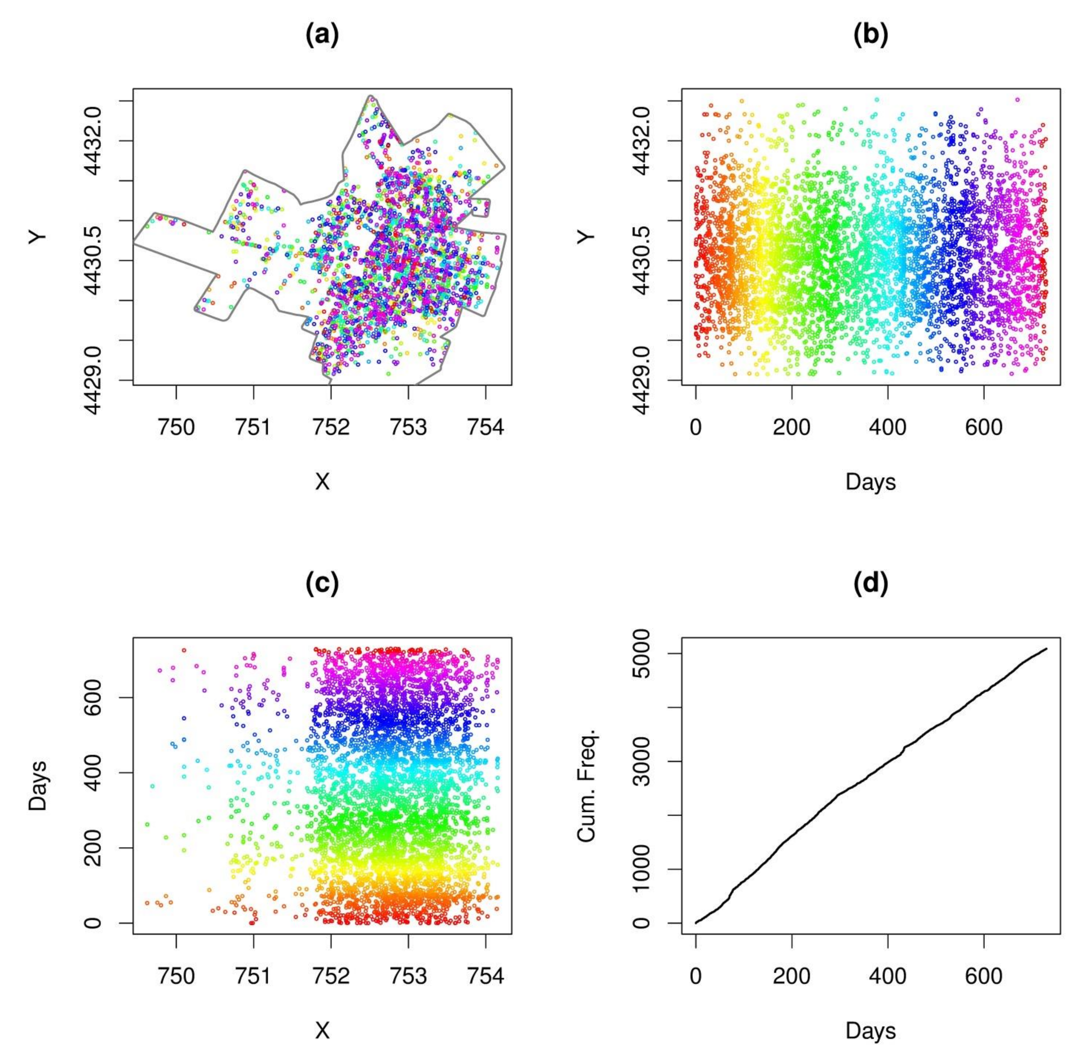

Figure 1. Basic information of the crime data: (a) Spatial locations, (b) y-t space-time plot, (c) t-x space-time plot, and (d) cumulative numbers verse occurrence times.

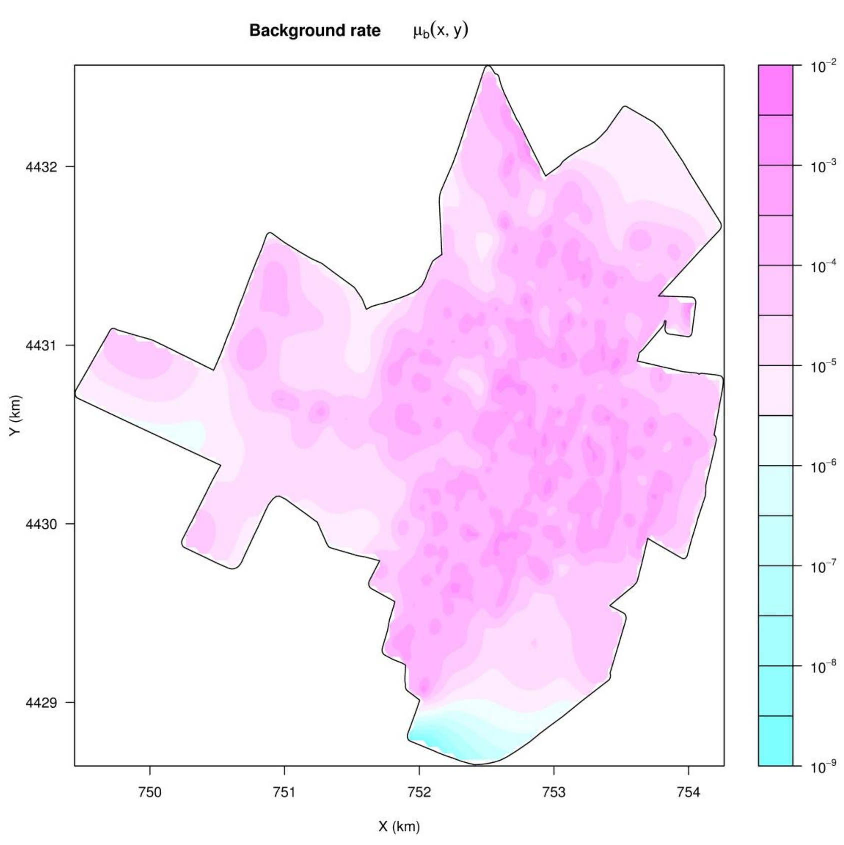

Figure 2. Estimated background rate (unit: vents/day/km^2).

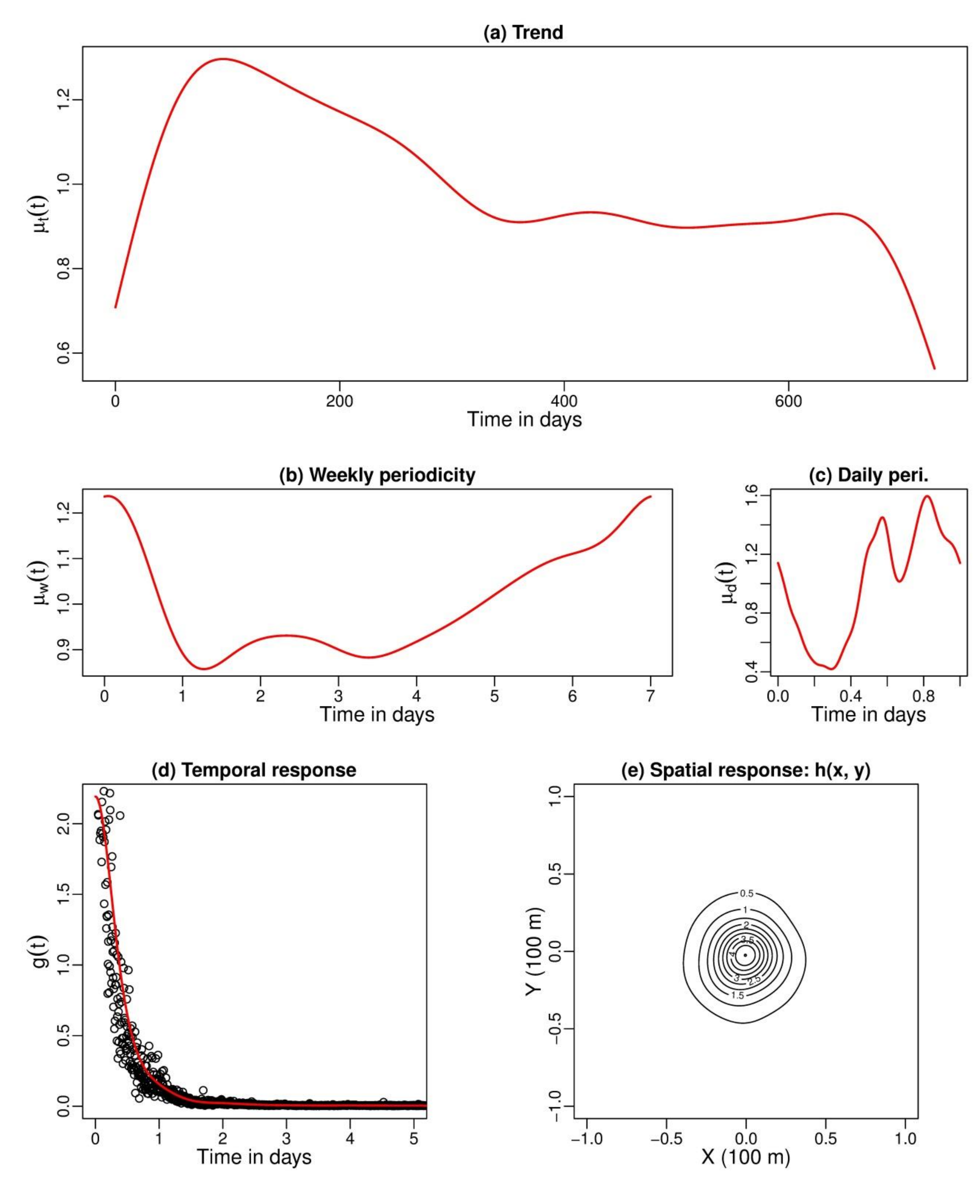

Figure 3. Output results: (a) trend function, (b) weekly periodicity, (c) daily periodicity, (d) temporal response function, and (e) spatial response function.

【References】

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