

# マーク付き点過程の欠損データのテスト及び補完

## Detection and replenishment of missing data for marked point processes

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**[Aim]** This project presents a fast approach for replenishing missing data in the record of a temporal point process with time separable marks, such as records of earthquakes and volcanic eruptions, in order to eliminate the underestimate of corresponding hazards caused by missing data.

### [What is a point process]

**Point process:** random patterns of discrete events in space and/or time.

**Marked point process:** each event is assigned an attribute or size, namely a mark.

**Conditional intensity:**  $\lambda(t, m | H_t) dt dm = \mathbf{E} [N([t, t + dt) \times (m, m + dm) | H_t)]$

**Mark-separable marked point process:**  $\lambda(t, m | H_t) = \lambda_g(t | H_t) f(m)$

$H_t$ : observation history up to time  $t$ .

### [Bi-scale empirical transformation]

**Bi-scale empirical transformation** transforms a mark-separable point process

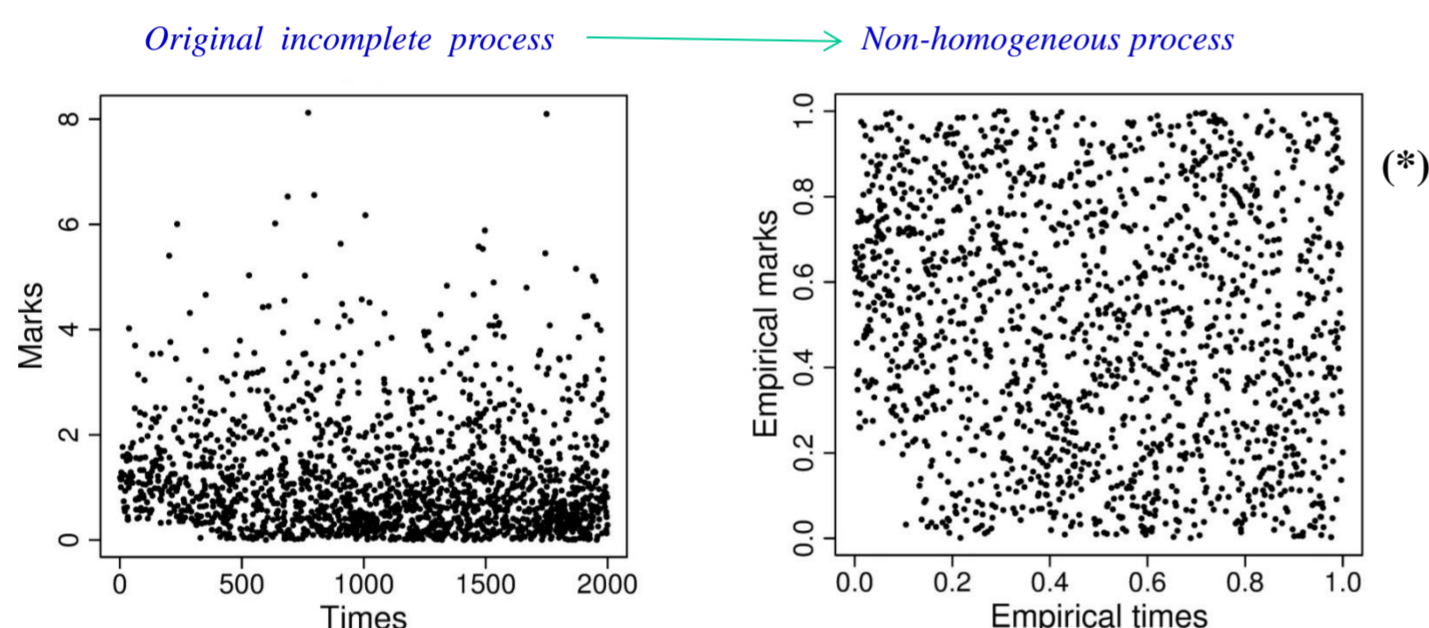
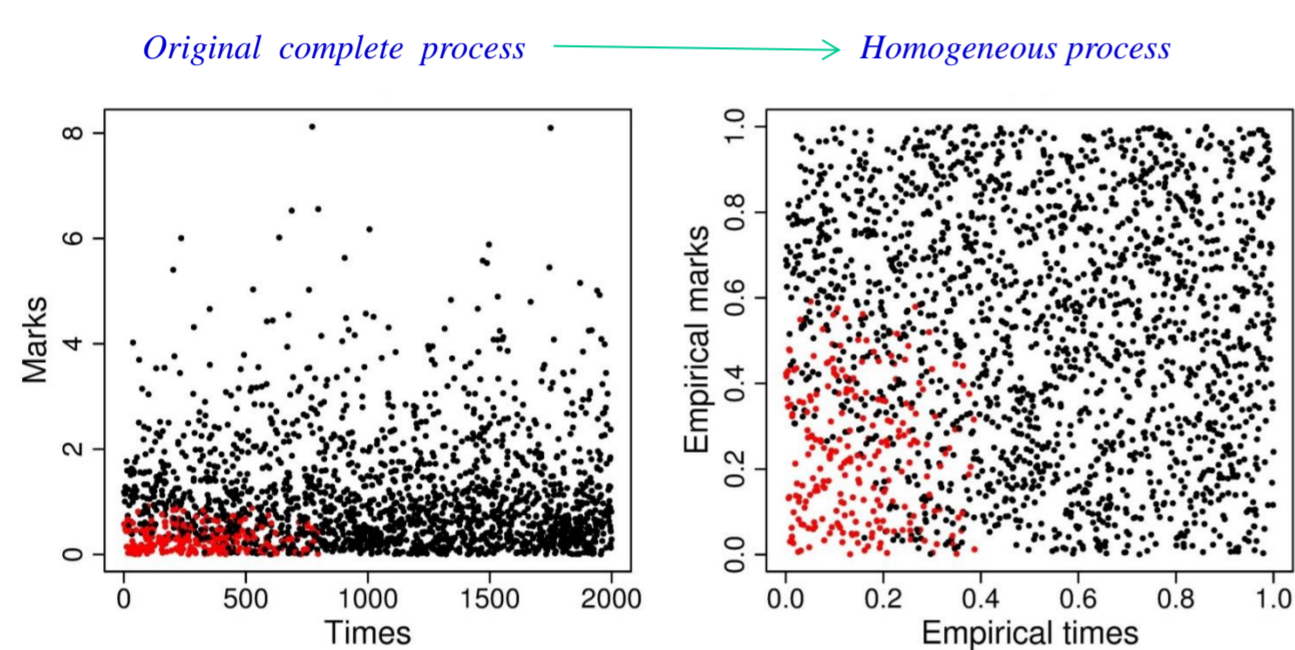
$N = \{(t_i, m_i) : i = 1, \dots, N\}$  on  $[0, T] \times \mathbf{M}$

into a homogeneous pattern on the unit square.

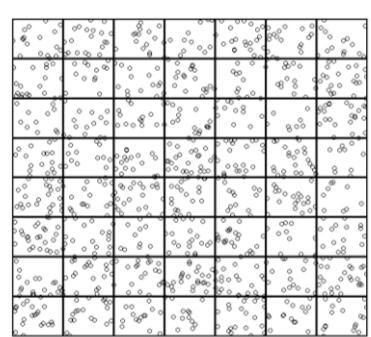
$$\Gamma: [0, T] \times \mathbf{M} \rightarrow [0, 1] \times [0, 1]$$

$$(t, m) \rightarrow (\tilde{F}(t), \tilde{G}(m))$$

$\tilde{F}(t), \tilde{G}(m)$ : empirical distribution function of  $\{t_i : i = 1, \dots, N\}$  and  $\{m_i : i = 1, \dots, N\}$



### [Testing methods]



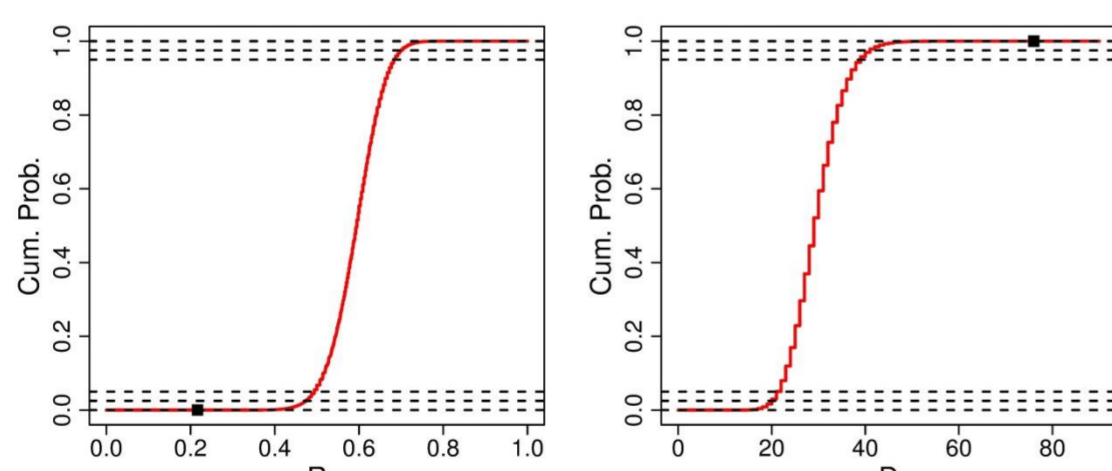
$$R = \frac{\min\{C_1, C_2, \dots, C_L\}}{\max\{C_1, C_2, \dots, C_L\}}$$

$$D = \max\{C_1, C_2, \dots, C_L\} - \min\{C_1, C_2, \dots, C_L\}$$

$$L = \#Row \times \#Column$$

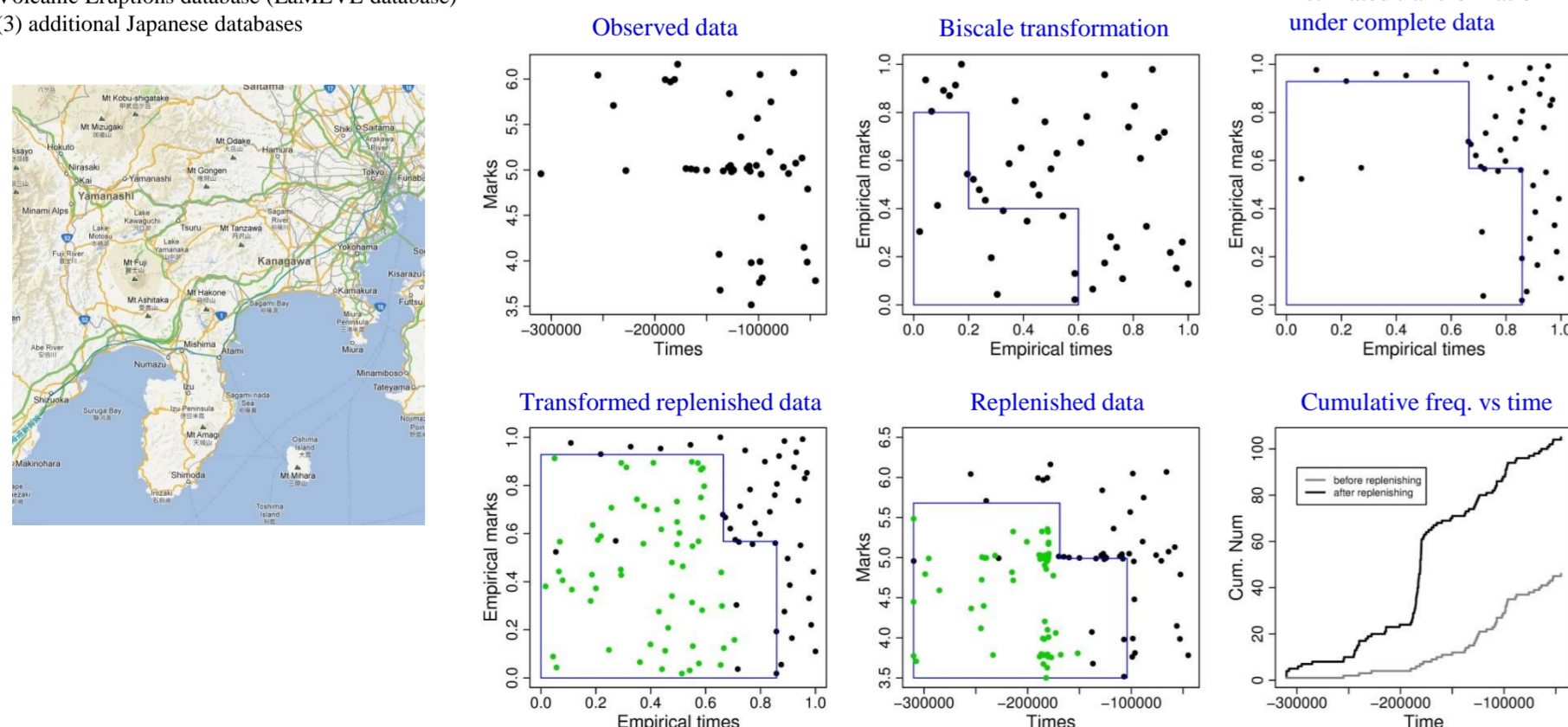
$C_i$ : number of events at each cells

Testing results for (\*)



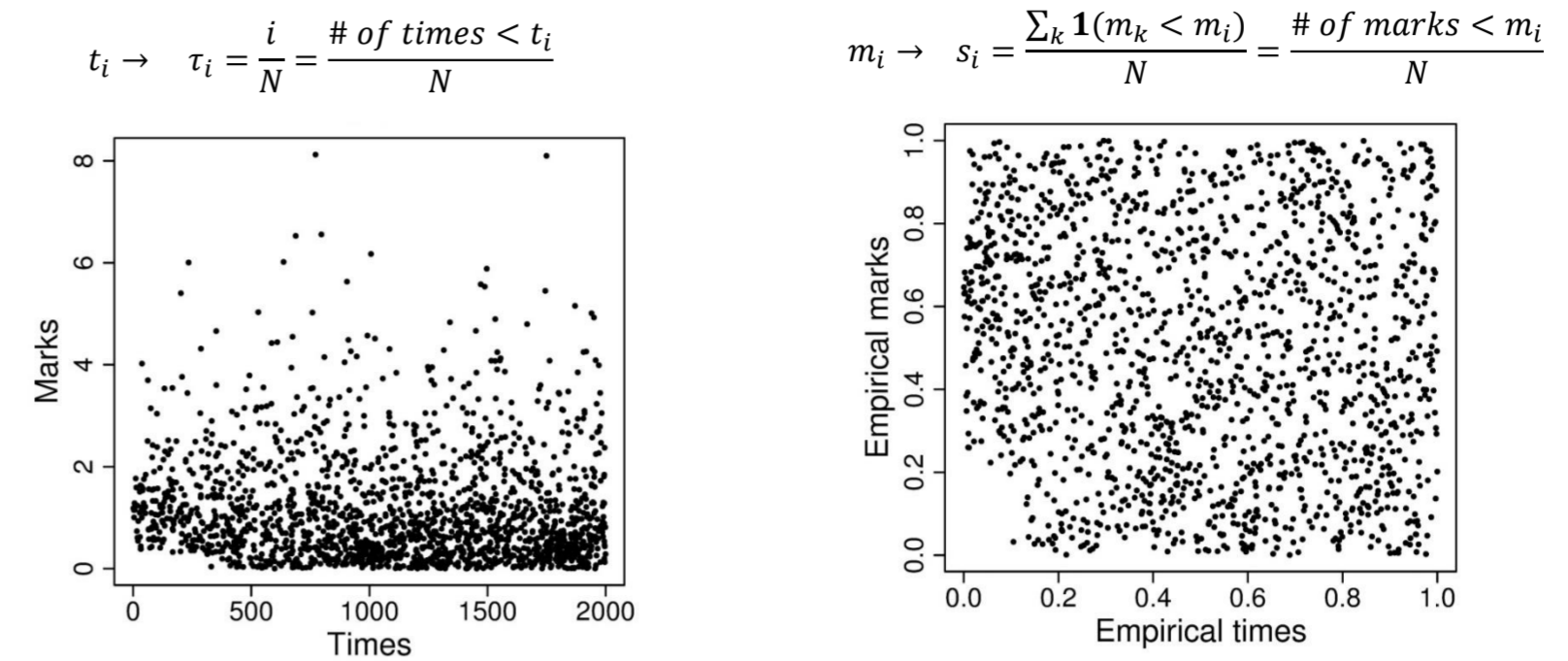
### [Example 1: Eruptions at Hakone volcano]

Data source: (1) Smithsonian's Global Volcanism Program database (2) Large Magnitude Explosive Volcanic Eruptions database (LaMEVE database) (3) additional Japanese databases

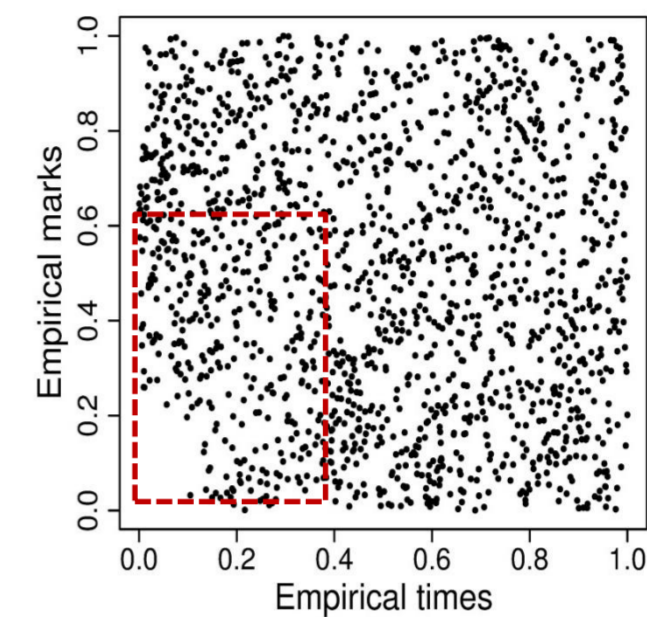


### [Replenish Algorithm]

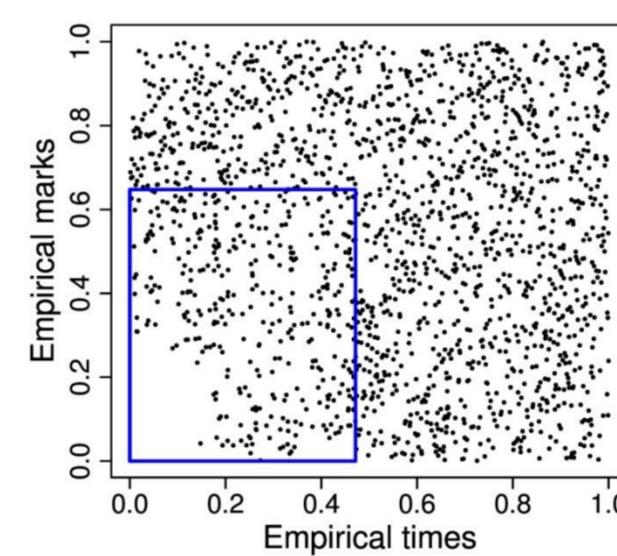
Step 1. Transform the process using the biscale empirical transformation.



Step 2. Specify area  $S$  that contains the missing data in the initial transformation domain.



Step 3. Calculate the missing area  $S^*$  in biscale transformation domain by solving



$$F_1^*(t) = \frac{\sum_{j=1}^n w_1(t_j, m_j, S) \mathbf{1}(t_j < t)}{\sum_{j=1}^n w_1(t_j, m_j, S)}$$

$$F_2^*(m) = \frac{\sum_{j=1}^n w_2(t_j, m_j, S) \mathbf{1}(m_j < m)}{\sum_{j=1}^n w_2(t_j, m_j, S)}$$

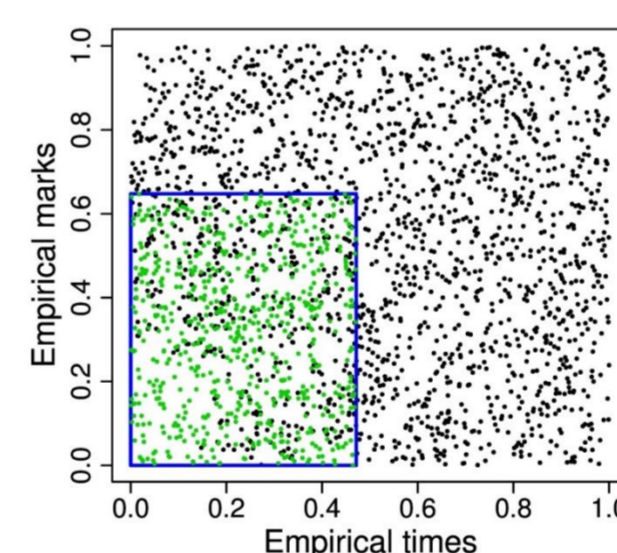
$$w_1(t, m, S) = \frac{\mathbf{1}((t, m) \notin S)}{\int_{\mathbf{M}} \mathbf{1}((t, s) \notin S) dF_2^*(s)}$$

$$w_2(t, m, S) = \frac{\mathbf{1}((t, m) \notin S)}{\int_{\mathbf{M}} \mathbf{1}((\tau, m) \notin S) dF_1^*(\tau)}$$

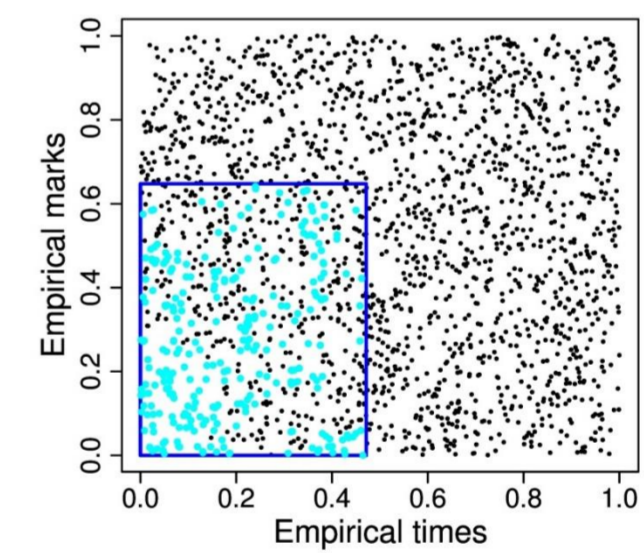
$$S^* = \{F_1^*(t), F_2^*(m) : (t, m) \in S\}$$

Step 4. Generate data point in the missing area

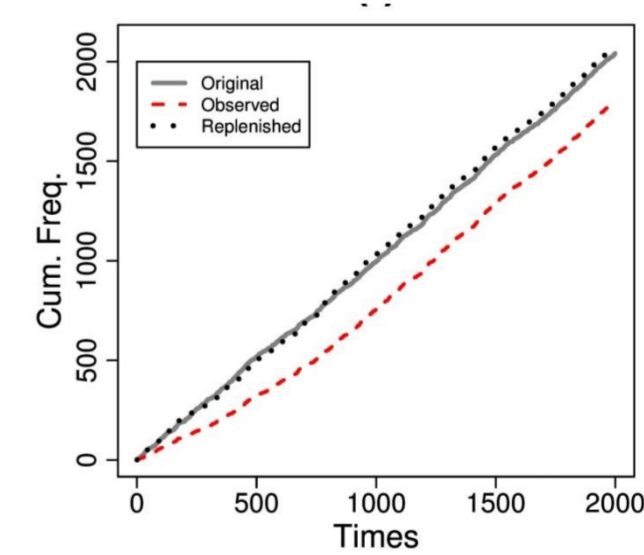
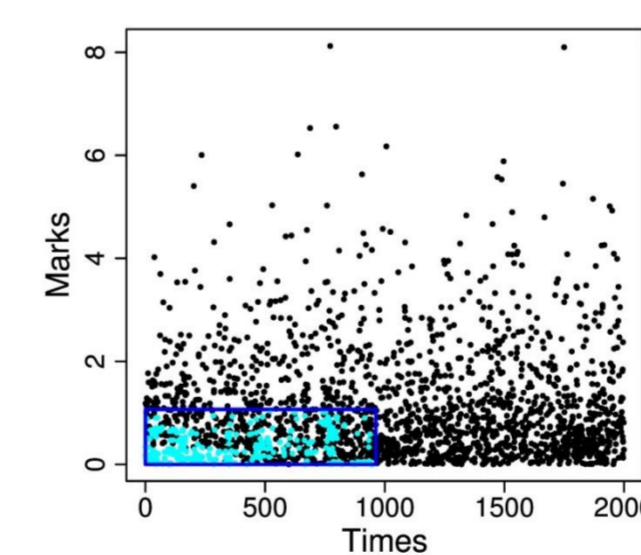
Generate events uniformly distributed in the missing region  $S^*$  (image of  $S$ )  
# events  $\sim NB(k, 1 - |S^*|)$   
 $k$ : # of observed events outside of  $S^*$



Step 5. Remove sequentially the closest simulated data point for each existing point in the missing area.



Step 5. Transform back all the events into the original domain



### [Example 2: Recent Kumamoto earthquake sequence data]

Data: Japan Meteorological Agency catalog  
Time: 2016/4/1-2016/4/21  
Mag.: 1.0+  
Depth: < 100 km  
Space: E128° - 133°  
N30° - 35°

