Cascading Failures by Fluctuating Loads in Scale-free Networks

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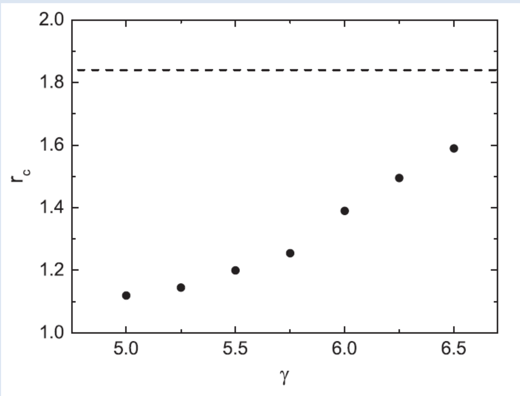
Topics

- Cascading failures on networks
- Network robustness
- Overload failures
- Fluctuating load
- Random walkers on a network

Conclusions

- The robustness of scale-free networks to cascading failures by temporally fluctuating loads has been studied for the first time.
- Cascading failures induced by fluctuating loads are formulated by the master equation and the generating function technique.
- The robustness of a network is measured by the critical load reduction parameter r_c above which the relative size S_f of the giant component at the final cascade stage is finite.
- Scale-free networks are robust against cascading overload failures in our model as opposed to previous works.
- The robustness of scale-free networks is explained by the property of the overload probability of being a decreasing function of the degree.



1. Introduction

Robustness of networks to failures

- Large-scale blackouts on power grids
- Communication failure in the Internet
- Spreading diseases
- etc.

= ***Percolation problem on networks***

Real-world failures

- Chain bankruptcy on a trading network
- Large-scale blackout by cascading failures of components in a power supply system
- etc.

Cascading overload failures on networks

(Failures induce further subsequent avalanche of failures.)

In functional networks

functionality ⟷ flow = load

Previous works

A. E. Motter and C.-Y. Lai, Phys. Rev. E **66**, 065102(R) (2002).
P. Holme and B.J. Kim, Phys. Rev. E **65**, 066109 (2002).
P. Crucitti, V. Latora, M. Marchiori, Phys. Rev. E **69**, 045104 (2004).
S.V. Buldyrev, R. Parshani, G. Paul, H.E. Stanley, and S. Havlin, Nature **464**, 1025 (2010).
and more ...

Constant flow through shortest paths

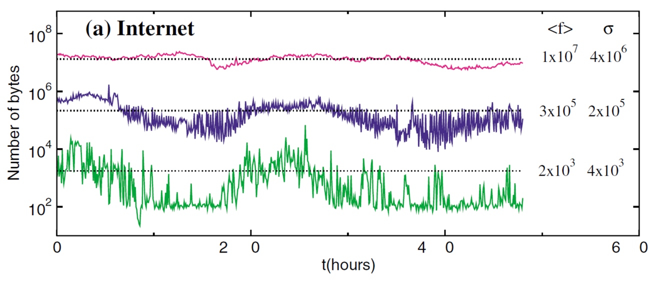
SF networks are fragile to cascading failures!

Fluctuating loads

Actual loads are temporally fluctuating.

load = average load + fluctuating load

instantaneous load value > node capacity



M. Argollo de Menezes and A.-L. Barabasi, PRL **92**, 028701 (2004).

overload failures

overload failure by fluctuating loads

➔ **subsequent avalanche of failures (cascade)**

Are SF networks still fragile to cascading failures by fluctuating loads?

2. Objective

To clarify how robust scale-free networks are against cascading failures by fluctuating loads.

3. Theory of overload failures

V. Kishore, M. S. Santhanam, and R. E. Amrittakar, PRL 106, 188701 (2011).

- Discretized loads ⇒ Random walkers
- Total load ⇒ Total number of random walkers (W_0)
- Load on node i ⇒ Number of random walkers on node i (w_i)
- Overload failures of node i ⇒ w_i exceeds the capacity q_i of i node

⇒ The overload probability is quantified under the above assumptions.

Overload probability

Random walk on a network J. D. Noh and H. Rieger, PRL **92**, 118701 (2004).

Distribution of the number of walkers on a degree-k node:

$$h_k^{W_0}(w) = \binom{W_0}{w} p_k{}^w (1-p_k)^{W_0-w}, \quad \text{where } p_k = \frac{k}{2M}$$

$$\langle w \rangle_k = p_k W_0 = \frac{W_0 k}{2M}, \quad \sigma_k^2 = \frac{W_0 k}{2M}\left(1-\frac{k}{2M}\right) \quad \text{depend only on } k$$

⇒ $q_k = \langle w \rangle_k + m\sigma_k$ Capacity

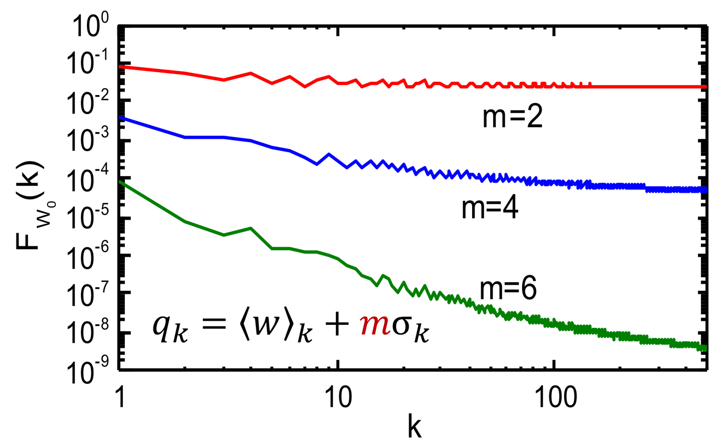
Overload probability $F_{W_0}(k)$ V. Kishore, M. S. Santhanam, and R. E. Amrittakar, PRL 106, 188701 (2011).

$$F_{W_0}(k) = \sum_{w=[q_k]+1}^{W_0} \binom{W_0}{w} p_k{}^w (1-p_k)^{W_0-w}$$

$$= I_{k/2M}([q_k(W_0)]+1, W_0 - [q_k(W_0)])$$

$I_p(a,b)$: Regularized incomplete beta function

$[x]$: The greatest integer not greater than x



Using the overload probability, we propose the cascading process below.

4. Cascade process

1. Remove nodes from an initial network $\mathcal{g}_0$ with N_0 nodes and M_0 edges according to the overload probability $F_{W_0}(k)$
2. At each time step τ, assign W_τ random walkers to the network $\mathcal{g}_\tau$. The total load W_τ is defined by the following relation:

$$W_\tau = \left(\frac{M_\tau}{M_0}\right)^r W_0 \qquad r: \text{load reduction parameter}$$

Examples:

- Injections of public money to markets or special low-interest lending facilities in trading networks
- Usage restriction of electricity to protect power-supply systems

3. Calculate the overload probability $F_{W_\tau}(k_0, k)$ of every node, and remove nodes from $\mathcal{g}_\tau$ with the probability $F_{W_\tau}(k_0, k)$.

$$F_{W_\tau}(k_0,k) = I_{k/2M_\tau}([q_{k_0}(W_0)]+1, W_\tau - [q_{k_0}(W_0)])$$

Determined at current time τ — Determined at initial time

4. Repeat procedures 2 and 3 until no node is removed.

5. Analytical approach

To formulate the cascade process by a master equation, we introduce following two quantities:

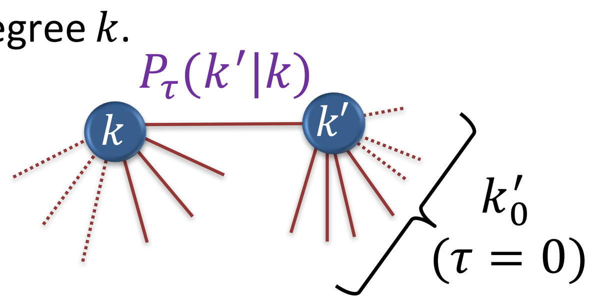
1. Extended degree distribution function $\Pi_\tau(k_0, k)$:

Probability that a randomly chosen node has the degree k at time τ and the initial degree k_0.

2. Overload probability of adjacent node $\phi_\tau(k)$:

Overload probability of an adjacent node of a node with degree k.

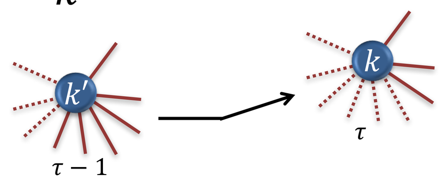
$$\phi_\tau(k) = \sum_{k_0'}^{k_{\max}} \sum_{k'=1}^{k_0'} P_\tau(k'|k) \frac{\Pi_\tau(k_0', k')}{P_\tau(k')} F_{W_\tau}(k_0', k')$$



Master equation for $\Pi_\tau(k_0', k')$

$$\Pi_\tau(k_0,k) = \sum_{k'=k}^{k_{\max}} \Pi_{\tau-1}(k_0,k')\left[\binom{k'}{k}\phi_{\tau-1}^{k'-k}(1-\phi_{\tau-1})^k\left[1-F_{W_{\tau-1}}(k_0,k')\right] + \delta_{k0}F_{W_{\tau-1}}(k_0,k')\right]$$

Time evolution of $\Pi_\tau(k_0', k')$



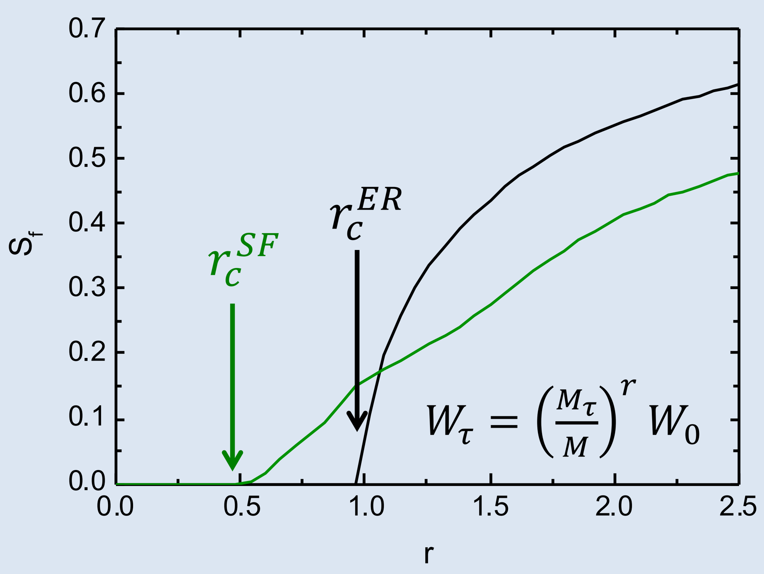
$\phi_\tau(k)$ ⟲ $\Pi_\tau(k_0, k)$ ⇒ $P_\tau(k) = \sum_{k_0} \Pi_\tau(k_0, k)$ ⇒ ***Size of giant component(GC) S_τ***

Generating function method

6. Results

- **Robustness of networks against cascading failures**

Load reduction parameter r dependence of GC size S_f at final stage



$r_c^{SF} < r_c^{ER}$

SF networks are ROBUST against cascading failures by fluctuating loads.

- **Consistency of our result**

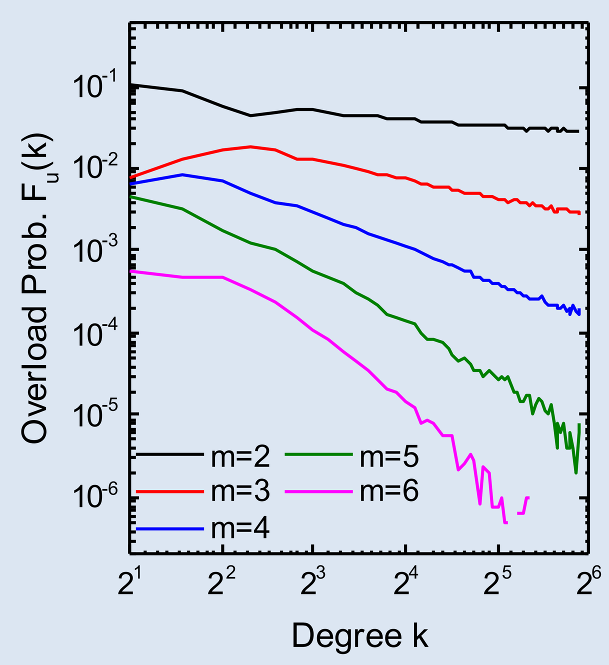
Shortest path flow model based on the *partial betweenness centrality* w_i

$$w_i = \sum_{(j,j')\in V_u} \frac{\sigma(j|i|j')}{\sigma(j|j')} \qquad V_u: \text{a set of } u \text{ node pairs} \quad 1 \ll u \ll N_0(N_0-1)/2$$

$$\{V_u^{(1)}, V_u^{(2)}, \cdots, V_u^{(n)}\} \rightarrow \tilde{h}_i(w)$$

$$\rightarrow \tilde{h}_k(w) \rightarrow q_k = \langle w \rangle_k + m\sigma_k$$

$$F_u(k) = \int_{q_k}^{\infty} \tilde{h}_k(w)\,dw$$



Overload probability is a decreasing function of degree k !