逐次モンテカルロ法による南極氷床コアの年代推定

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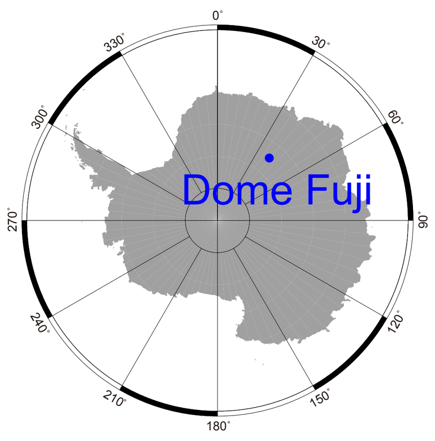
Motivation

- Ice cores provide vital information on the climatic and environmental changes over the past hundreds of thousands of years.
- In order to make use of the chronological records from ice cores, it is crucial to obtain an accurate estimate of the relationship between age and depth in the ice cores.



The age-depth relationship is mainly dependent on
→ the accumulation of snow at the site of the ice core,
→ and the thinning process due to the horizontal stretching and vertical compression of an ice layer.

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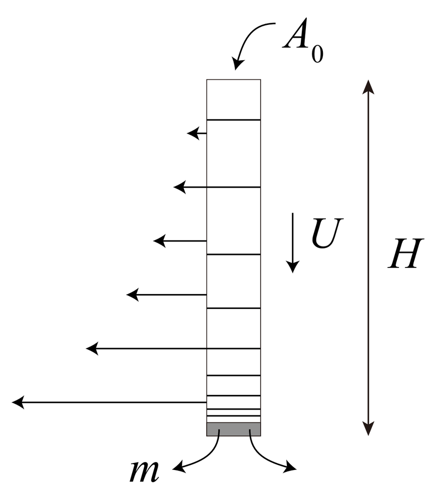
The accumulation of snow and the thinning factor are also estimated simultaneously with the age as a function of depth.



Formulation for the age-depth relationship

We consider the following differential equation describing the age-depth relationship:

$$dz = A(z)\Theta(z)d\xi$$

z: vertical coordinate for depth from the surface of the ice sheet,
ξ: age in year (past is positive),
$A(z)$: the snow accumulation rate per year,
$\Theta(z)$: thinning factor due to the long-term deformation within the ice sheet.

The age can be obtained by the following integral:

$$\xi(z) = \int_S^z \frac{dz'}{A(z')\Theta(z')},$$

where S indicates the surface of the ice sheet. Thus, this integral means that the age ξ is obtained by the integral from the surface.

Assuming a steady state, the thinning function $\Theta(z)$ is written using the vertical flow U:

$$\Theta(z) = U(z)/U(0).$$

We rescale z and U as follows

$$\zeta = \frac{H-z}{H}, \quad u(\zeta) = -\frac{U(z)}{H},$$

and write the rescaled flow u in the following form:

$$u(\zeta) = u(0) + [u(1) - u(0)]\omega(z) = -\frac{1}{H}[m + (A_0 - m)\omega(z)],$$

where A_0 denotes the accumulation rate at the surface and m denotes the melting at the base of ice sheet. Then, Θ can be rewritten as:

$$\Theta(\zeta) = \frac{\omega(\zeta)+\mu}{1+\mu}, \quad (\mu = m/[A_0 - m]).$$

Accroding to Parrenin et al. (2007)*, ω is assumed to be written in the following form:

$$\omega(\zeta) = \zeta - \frac{1-s}{1+p}(1-\zeta)\left[1-(1-\zeta)^{1+p}\right]$$

*Parrenin, F., et al. (2007): 1-D-ice flow modelling at EPICA Dome C and Dome Fuji, East Antarctica, Clim. Past, v. 3, p. 243.

Sequential Bayesian model

Sequential model describing the vertical chain

The vertical profile of the age ξ is modeled using the following recurrence equations:

$$\xi_{z+1} = \xi_z + \frac{1}{A_z\Theta_z} + \nu_z, \quad \log A_{z+1} = \log A_z + \eta_z,$$

where
ξ_z: the age at z;
Z: the depth at the bottom end of the ice core;
A_z: the accumulation rate in the interval from z to $z+1$;
Θ_z: the thinning factor in the interval from z to $z+1$;
ν_z: variation of the age due to unknown processes;
η_z: the variation of the accumulation rate:
(Note that the transition of A_z is described using its logarithm in order to guarantee $A_z > 0$.)

We combine ξ_z and A_z into one vector $\boldsymbol{x}_z$. The relationship between $\boldsymbol{x}_{z-1}$ and $\boldsymbol{x}_z$ can thus be written as $p(\boldsymbol{x}_z \mid \boldsymbol{x}_{z-1}, \theta)$ according to the above equations.

Available data and relationship between model variables and data

In order to estimate ξ_z and A_z for each z, we refer to two kinds of data:

$\delta^{18}O(z)$: a proxy of the temperature around the site, which is related with the accumulation rate,
Age markers: other reliable proxies at several depth levels (Kawamura et al., 2007)*.

We assume the following relationship between an age marker τ_k and the modeled age ξ_{z_k}:

$$\tau_k = \xi_{z_k} + \varepsilon_k,$$

and the following relationship is assumed between the accumulation rate A_z and $\delta^{18}O(z)$:

$$\delta^{18}O_z = \alpha \log A_z + \beta + w_z.$$

We combine τ_z and $\delta^{18}O_z$ into one vector $\boldsymbol{y}_z$ and write the relationship between $\boldsymbol{x}_z$ and $\boldsymbol{y}_z$ as $p(\boldsymbol{y}_z \mid \boldsymbol{x}_z, \theta)$.

*Kawamura, K., et al. (2007): Northern Hemisphere forcing of climatic cycles in Antarctica over the past 360,000 years, Nature, v. 448, p. 912.

Particle Markov chain Monte Carlo method

An estimate of the age as a function of depth can be obtained from the posterior distribution after θ are marginalised out:

$$p(\boldsymbol{x}_{0:Z} \mid \boldsymbol{y}_{1:Z}) = \int p(\boldsymbol{x}_{0:Z} \mid \boldsymbol{y}_{1:Z}, \theta)\, p(\theta \mid \boldsymbol{y}_{1:Z})\, d\theta.$$

Assuming that $\boldsymbol{y}_z$ is conditionally independent of $\boldsymbol{x}_{z'}$ given $\boldsymbol{x}_z$,

$$p(\boldsymbol{x}_{0:z} \mid \boldsymbol{y}_{1:z}, \theta) \propto p(\boldsymbol{y}_z \mid \boldsymbol{x}_z, \theta)\, p(\boldsymbol{x}_z \mid \boldsymbol{x}_{z-1}, \theta)\, p(\boldsymbol{x}_{0:z-1} \mid \boldsymbol{y}_{0:z-1}, \theta)$$

We can obtain a set of samples from $p(\boldsymbol{x}_{0:z} \mid \boldsymbol{y}_{1:z}, \theta)$ given θ using a sequential Monte Carlo method.

The posterior distribution of θ given $\boldsymbol{y}_z$ can be calculated using the following equation:

$$p(\theta \mid \boldsymbol{y}_{1:Z}) \propto p(\boldsymbol{y}_{1:Z} \mid \theta)\, p(\theta)$$

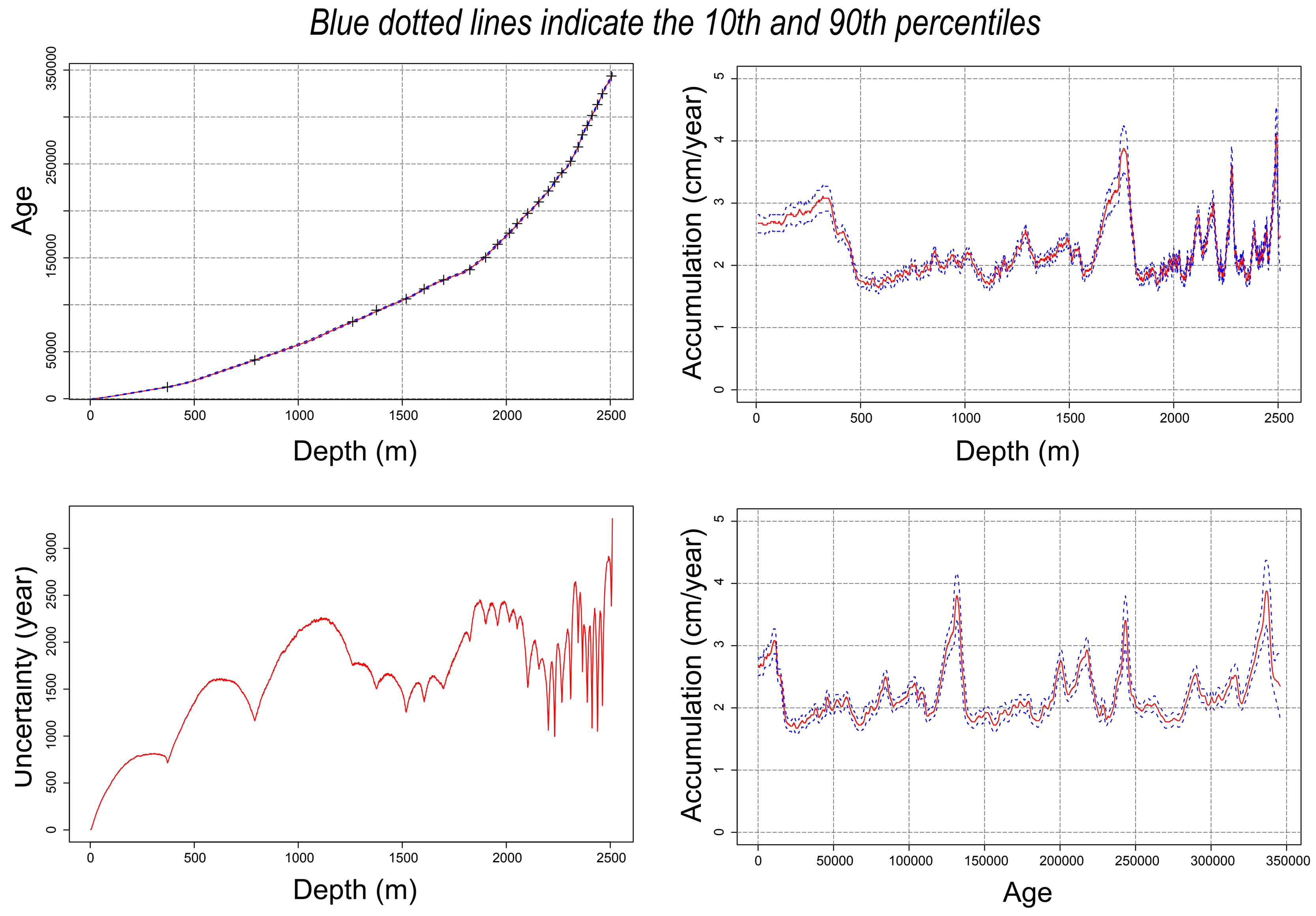
Samples from $p(\boldsymbol{y}_{1:Z} \mid \theta)$ can be obtained using the sequential Monte Carlo method. Therefore, samples from $p(\theta \mid \boldsymbol{y}_{1:Z})$ can be drawn using the Metropolis method.

At each Metropolis iteration for generating a sample from $p(\theta \mid \boldsymbol{y}_{1:Z})$, we can draw samples from $p(\boldsymbol{x}_{0:Z} \mid \boldsymbol{y}_{1:Z}, \theta)$ using the sequential Monte Carlo method. Collecting the samples for various θ, we can obtain a set of samples obeying the marginal posterior $p(\boldsymbol{x}_{0:Z} \mid \boldsymbol{y}_{1:Z})$. (Andrieu et al., 2010)*.

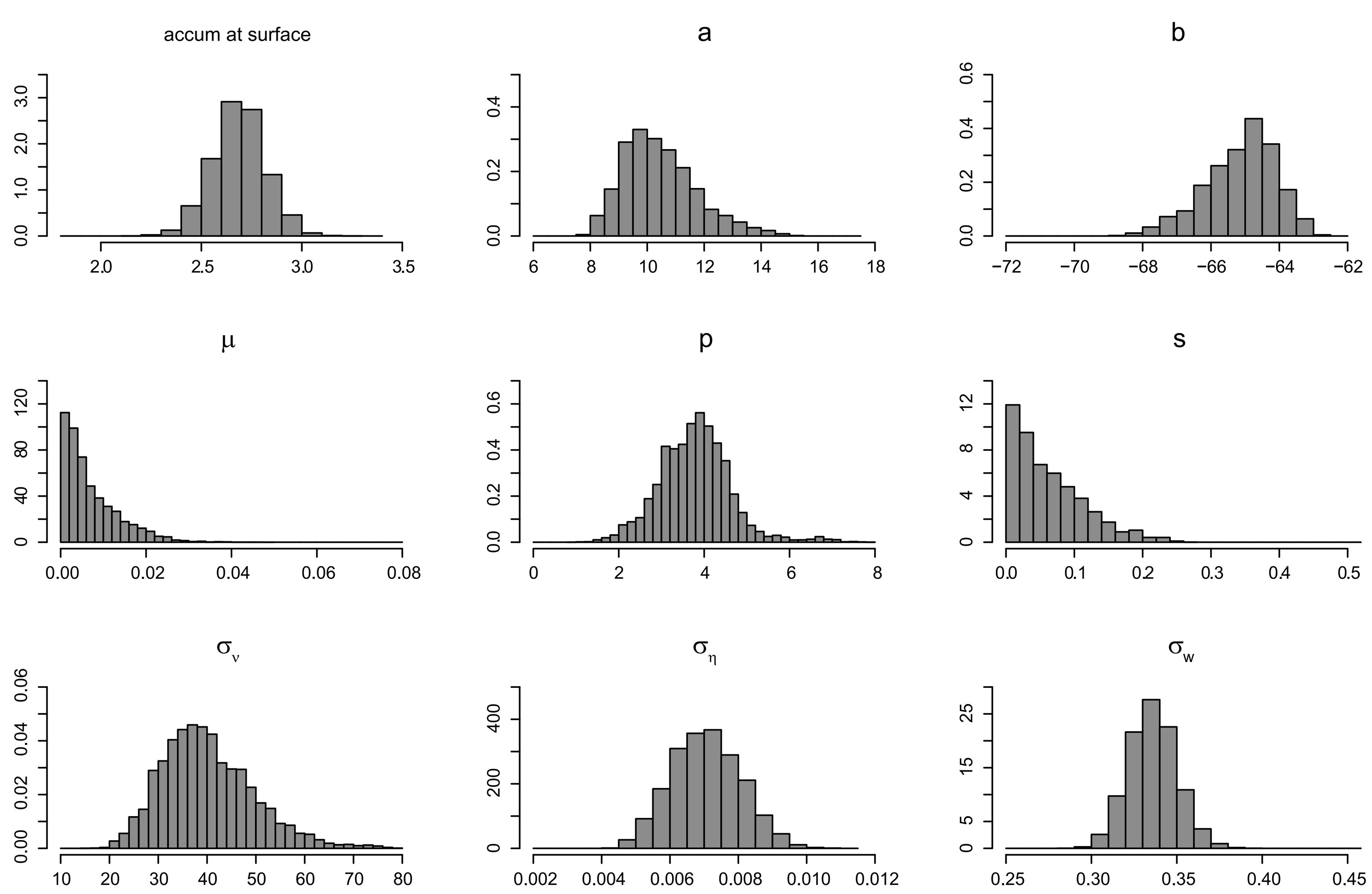
*Andrieu, C., A. Doucet, and R. Holenstein (2010): Particle Markov chain Monte Carlo methods, J. Roy. Statist. Soc. B, v. 72, p. 269.

Result

Blue dotted lines indicate the 10th and 90th percentiles



(The uncertainty was evaluated with the difference between the 10th and 90th percentiles of the posterior.)



Marginal posterior distributions for 9 parameters