(16) TABLES OF THE POWER FUNCTION OF ANALYSIS OF VARIANCE TEST

Taken from P.C. Tang, "The Power Function of the Analysis of Variance Tests with Tables and Illustrations of Their Use". Stat. Res. Mem. 2: 126-157. 1938.

Directions for use of the tables *:

In the following tables there appears the quantity E^2 which may be used in testing the null hypothesis that no difference exists among treatments. The test may be made at significance levels of 0.01 (table I) and 0.05 (table II) as indicated by the subscripts of E^2. The null hypothesis is rejected if the observed E^2 is greater than the appropriate tabulated value.

Tang used this criterion rather than Snedecor's "F" or Fisher's "Z" apparently because it simplified

the computation of the power function. E^2 is related to "F" as follows:

$$E^2 = \frac{f_1 F}{f_2 + f_1 F} \tag{1}$$

where f_1 = degree of freedom for the treatment mean square

f_2 = degree of freedom for the error mean square.

Since $$F = \frac{S_1^2/f_1}{S_2^2/f_2} \tag{2}$$

where S_1^2 = treatment sum of squares

S_2^2 = error sum of squares.

Substituting (2) in (1) yields

$$E^2 = \frac{S_1^2}{S_2^2 + S_1^2} \tag{3}$$

The symbol P_{II} * in the tables refers to the probability of making a Type II error, and the quantities f_1 and f_2 are defined as above.

The quantity ϕ is defined as

$$\phi = \sqrt{\frac{2\lambda}{K}} \tag{4}$$

where

$$\lambda = \frac{n}{2\sigma^2} \sum_{i=1}^{k} \theta_i^2 \tag{5}$$

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* Some of the notation here used is slightly different from that employed by Tang. (P-131) * The power = $1 - P_{II}$. (P-132)

with

k = number of treatments.

n = number of replications.

σ^2 = expected error mean square (variance of the experimental unit).

θ_i = true effect of the ith treatment when the origin is placed at the true general mean.

$$\therefore \quad \sum_{i=1}^{k} \theta_i = 0$$

Substituting (5) in (4) yields

$$\phi = \frac{\sqrt{n}}{\sigma} \sqrt{\frac{\sum_{i=1}^{k} \theta_i^2}{k}} \qquad (6)$$

Thus ϕ is the ratio of the standard deviation of the true treatment effects to the standard error of a treatment mean.

Example of application of (6):

Suppose it is desired to study four treatments ($k=4$), each with five replications ($n=5$), in a randomized block experiment, and the true treatment effects, θ_i, are $-5, -4, 3$ and 6. From past experience it appears that σ is about 10. Then from (6)

$$\phi = \frac{\sqrt{5}}{10}\sqrt{\frac{25+16+9+36}{4}}$$

$$= 1.04$$

Referring to table II where $P_I = 0.05$, $f_1 = 3$, $f_2 = 12$ and $\phi = 1.04$ we find that P_{II} equals about 0.7. In other words, the combination of true treatments effects given above would be found significant at the 5 % level in only 3 experiments out of 10. There are, of course an infinite number of sets of σ_i which can yield $\phi = 1.04$, or any other value of ϕ for that matter.

The power function of the "t" test:

Neyman and collaborators (J. Royal Stat. Soc. Suppl. 2:107, 1935; and J. Amer. Stat. Assoc. 31:318, 1936) have published tabulations of the power function of the one-tailed "t" test. They used ρ as the argument analagous to ϕ in the present tables, with

$$\rho = \frac{\Delta}{\sigma_\Delta}$$

where Δ = the true difference between treatments.

σ = the true error of the difference.

In the special case where $f_1 = 1$, it may be shown

that

$$\rho = \phi\sqrt{2}$$

Using this relation the power of the two-tailed "t" test for a given ρ may be found in the following tables where $f_1 = 1$.

Table I. Table of $E^2_{0.01}$ and the corresponding values of P_{II}

$f_1 = 1$

f_2	$E^2_{0.01}$	ϕ									
		1	1.5	2	2.5	3	4	5	6	7	8
2	.980	.970	.947	.914	.874	.828	.720	.602	.484	.373	.277
4	.841	.949	.885	.784	.651	.501	.233	.077	.018	.003	
6	.696	.934	.839	.687	.498	.312	.076	.010	.001		
7	.636	.928	.822	.652	.447	.258	.049	.006			
8	.585	.924	.808	.624	.409	.221	.034	.002			
9	.540	.920	.796	.601	.379	.193	.025	.001			
10	.501	.916	.786	.582	.355	.172	.019	.001			
11	.467	.913	.777	.567	.336	.156	.015				
12	.437	.911	.770	.553	.320	.144	.012				
13	.411	.909	.763	.542	.307	.133	.010				
14	.388	.907	.758	.532	.296	.125	.009				
15	.367	.905	.753	.523	.286	.118	.008				
16	.348	.904	.749	.516	.278	.112	.007				
17	.331	.902	.745	.509	.271	.107	.006				
18	.315	.901	.741	.503	.264	.103	.006				
19	.301	.900	.738	.498	.259	.099	.005				
20	.288	.899	.735	.493	.254	.096	.005				
21	.276	.898	.732	.488	.249	.093	.004				
22	.265	.897	.730	.484	.245	.090	.004				
23	.255	.896	.728	.481	.241	.088	.004				
24	.246	.896	.726	.477	.238	.086	.004				
25	.237	.895	.724	.474	.235	.084	.004				
26	.229	.894	.722	.471	.232	.082	.003				
27	.221	.894	.720	.469	.229	.081	.003				
28	.214	.893	.718	.466	.227	.079	.003				
29	.212	.893	.717	.464	.225	.078	.003				
30	.201	.892	.716	.462	.223	.077	.003				
60	.106	.885	.696	.430	.194	.061	.002				
∞		.877	.675	.400	.169	.048	.001				

Table I. Table of $E^2_{0.01}$ and the corresponding values of P_{II}

$f_1 = 2$

f_2	$E^2_{0.01}$	ϕ									
		1	1.5	2	2.5	3	4	5	6	7	8
2	.990	.975	.957	.932	.901	.865	.779	.680	.577	.475	.379
4	.900	.957	.901	.810	.685	.540	.266	.095	.024	.004	.001
6	.785	.941	.850	.695	.498	.305	.068	.007			
7	.732	.934	.828	.649	.437	.235	.035	.004			
8	.684	.929	.809	.611	.379	.187	.021	.001			
9	.681	.924	.793	.579	.338	.152	.013				
10	.602	.920	.779	.552	.306	.127	.008				
11	.567	.916	.767	.528	.278	.108	.006				
12	.536	.912	.756	.508	.255	.093	.005				
13	.508	.909	.746	.491	.237	.082	.003				
14	.482	.907	.738	.476	.223	.074	.002				
15	.459	.904	.730	.463	.211	.066	.002				
16	.438	.902	.723	.452	.201	.060	.001				
17	.418	.900	.717	.442	.193	.055	.001				
18	.401	.898	.711	.443	.185	.051	.001				
19	.384	.896	.706	.424	.177	.048	.001				
20	.369	.895	.701	.417	.170	.045	.001				
21	.355	.894	.697	.410	.165	.042	.001				
22	.342	.893	.693	.404	.160	.040	.001				
23	.330	.891	.690	.399	.155	.038					
24	.319	.890	.686	.394	.151	.036					
25	.308	.889	.683	.389	.148	.035					
26	.298	.888	.680	.385	.144	.034					
27	.289	.887	.678	.381	.141	.032					
28	.280	.886	.675	.377	.138	.031					
29	.272	.886	.672	.373	.136	.030					
30	.264	.885	.670	.370	.134	.029					
60	.142	.873	.637	.324	.102	.019					
∞		.860	.601	.279	.076	.011					

Table I. Table of $E^2_{0.01}$ and the corresponding values of P_{II}

$f_1 = 3$

f_2	$E^2_{0.01}$	ϕ									
		1	1.5	2	2.5	3	4	5	6	7	8
2	.993	.977	.961	.939	.911	.878	.800	.709	.612	.515	.421
4	.926	.959	.907	.818	.695	.552	.276	.100	.026	.005	.001
6	.830	.943	.850	.691	.486	.290	.059	.006			
7	.784	.936	.825	.636	.408	.210	.025	.002			
8	.740	.929	.803	.590	.347	.158	.014				
9	.700	.923	.783	.550	.299	.120	.008				
10	.663	.918	.765	.517	.261	.094	.004				
11	.629	.913	.749	.487	.231	.075	.002				
12	.598	.909	.735	.463	.206	.062	.001				
13	.570	.906	.723	.441	.186	.051	.001				
14	.544	.902	.711	.422	.170	.044	.001				
15	.520	.899	.701	.406	.156	.038	.001				
16	.498	.896	.692	.391	.145	.033					
17	.478	.893	.683	.378	.135	.029					
18	.459	.891	.676	.367	.126	.026					
19	.442	.889	.669	.356	.119	.023					
20	.426	.887	.662	.347	.112	.021					
21	.410	.885	.656	.339	.107	.019					
22	.396	.883	.651	.331	.102	.017					
23	.383	.881	.646	.324	.098	.016					
24	.371	.880	.641	.318	.094	.015					
25	.359	.879	.637	.312	.090	.014					
26	.349	.877	.633	.307	.087	.013					
27	.338	.876	.629	.302	.084	.012					
28	.329	.875	.625	.297	.081	.012					
29	.319	.874	.622	.293	.079	.011					
30	.311	.872	.619	.289	.077	.011					
60	.171	.856	.571	.233	.050	.005					
∞		.836	.519	.182	.030	.002					

Table I. Table of $E^2_{0.01}$ and the corresponding values of P_{II}

$f_1 = 4$

f_2	$E^2_{.001}$	ϕ									
		1	1.5	2	2.5	3	4	5	6	7	8
2	.995	.978	.962	.942	.915	.884	.810	.724	.631	.536	.444
4	.941	.960	.909	.822	.700	.557	.280	.102	.027	.005	.001
6	.859	.943	.849	.685	.475	.277	.053	.005			
7	.818	.936	.821	.624	.389	.191	.018				
8	.778	.928	.796	.571	.322	.136	.010				
9	.741	.922	.773	.526	.269	.098	.003				
10	.706	.916	.752	.487	.227	.073	.002				
11	.673	.911	.733	.453	.195	.055	.001				
12	.643	.906	.716	.424	.169	.042	.001				
13	.616	.901	.700	.398	.148	.034					
14	.590	.897	.687	.376	.131	.028					
15	.566	.893	.674	.357	.117	.022					
16	.544	.890	.662	.340	.106	.018					
17	.523	.886	.652	.325	.096	.015					
18	.504	.883	.642	.312	.088	.013					
19	.486	.880	.633	.301	.081	.011					
20	.470	.878	.625	.290	.075	.010					
21	.454	.876	.618	.280	.070	.009					
22	.440	.873	.611	.272	.066	.008					
23	.426	.871	.604	.264	.062	.007					
24	.413	.869	.598	.257	.059	.006					
25	.401	.867	.593	.250	.056	.006					
26	.389	.865	.588	.244	.053	.005					
27	.378	.864	.583	.239	.050	.005					
28	.368	.862	.578	.234	.048	.005					
29	.358	.861	.574	.229	.046	.004					
30	.349	.860	.570	.225	.044	.004					
60	.196	.837	.509	.165	.024	.001					
∞		.810	.443	.115	.011						

Table I. Table of $E^2_{0.01}$ and the corresponding values of P_{II}

$f_1 = 5$

f_2	$E^2_{0.01}$	ϕ									
		1	1.5	2	2.5	3	4	5	6	7	8
2	.996	.978	.964	.944	.918	.888	.817	.733	.642	.549	.458
4	.951	.961	.910	.824	.702	.559	.282	.103	.027	.005	.001
6	.879	.943	.848	.679	.466	.266	.048	.004			
7	.842	.935	.818	.614	.394	.177	.014				
8	.806	.928	.790	.556	.301	.121	.007				
9	.771	.920	.764	.505	.245	.083	.003				
10	.738	.914	.740	.461	.201	.059	.001				
11	.707	.908	.718	.424	.168	.042					
12	.679	.902	.699	.391	.141	.031					
13	.652	.897	.681	.363	.120	.023					
14	.626	.892	.664	.339	.104	.018					
15	.603	.888	.649	.318	.090	.014					
16	.581	.883	.636	.299	.079	.011					
17	.561	.880	.624	.283	.071	.009					
18	.541	.876	.612	.269	.063	.007					
19	.523	.873	.602	.256	.057	.006					
20	.506	.870	.592	.245	.052	.005					
21	.490	.867	.583	.234	.047	.004					
22	.475	.864	.575	.225	.044	.004					
23	.461	.861	.567	.217	.040	.003					
24	.448	.859	.560	.210	.037	.003					
25	.435	.857	.553	.203	.035	.003					
26	.423	.855	.547	.196	.033	.002					
27	.412	.853	.541	.190	.031	.002					
28	.401	.851	.536	.185	.029	.002					
29	.391	.849	.531	.180	.027	.002					
30	.381	.847	.526	.176	.026	.002					
60	.218	.819	.452	.116	.017						
∞		.784	.373	.070	.004						

Table I. Table of $E^2_{0.01}$ and the Corresponding values of P_{II}

$f_1 = 6$

f_2	$E^2_{0.01}$	ϕ									
		1	1.5	2	2.5	3	4	5	6	7	8
2	.997	.978	.964	.945	.920	.891	.821	.739	.650	.558	.468
4	.958	.962	.911	.825	.704	.560	.283	.104	.027	.005	.001
6	.894	.944	.847	.675	.459	.258	.044	.003			
7	.860	.935	.815	.605	.362	.166	.011				
8	.827	.927	.784	.543	.285	.109	.006				
9	.795	.919	.756	.488	.226	.071	.003				
10	.764	.912	.730	.441	.181	.048	.001				
11	.734	.905	.706	.400	.147	.033					
12	.707	.899	.683	.365	.120	.023					
13	.681	.893	.663	.334	.100	.017					
14	.656	.888	.645	.308	.084	.013					
15	.633	.882	.628	.286	.071	.009					
16	.612	.878	.612	.266	.061	.007					
17	.591	.873	.598	.249	.053	.005					
18	.572	.869	.585	.233	.046	.004					
19	.554	.865	.573	.220	.041	.003					
20	.537	.862	.562	.208	.036	.003					
21	.521	.858	.552	.198	.033	.002					
22	.506	.855	.542	.188	.029	.002					
23	.492	.852	.533	.180	.027	.002					
24	.478	.849	.524	.172	.024	.001					
25	.465	.846	.517	.165	.022	.001					
26	.453	.844	.510	.159	.020	.001					
27	.442	.842	.503	.153	.019	.001					
28	.430	.839	.497	.147	.017	.001					
29	.420	.837	.491	.142	.016	.001					
30	.410	.835	.486	.138	.015	.001					
60	.238	.801	.401	.081	.006						
∞		.755	.311	.042	.001						

Table I. Table of $E^2_{0.01}$ and the corresponding values of P_{II}

$f_1 = 7$

f_2	$E^2_{0.01}$	ϕ									
		1	1.5	2	2.5	3	4	5	6	7	8
2	.997	.979	.965	.946	.922	.893	.824	.743	.655	.564	.475
4	.963	.962	.912	.826	.705	.561	.283	.104	.027	.005	.001
6	.906	.944	.845	.671	.452	.251	.041	.003			
7	.875	.935	.812	.598	.351	.158	.009				
8	.844	.926	.779	.532	.272	.100	.005				
9	.814	.918	.749	.474	.211	.063	.002				
10	.785	.910	.720	.423	.166	.041	.001				
11	.757	.903	.694	.379	.131	.027					
12	.730	.896	.670	.342	.105	.018					
13	.705	.889	.648	.310	.085	.013					
14	.681	.883	.627	.283	.069	.009					
15	.659	.878	.608	.259	.057	.007					
16	.638	.872	.591	.238	.048	.004					
17	.618	.868	.575	.220	.041	.003					
18	.599	.863	.561	.205	.035	.002					
19	.581	.859	.548	.191	.030	.002					
20	.564	.854	.535	.179	.026	.002					
21	.548	.851	.524	.168	.023	.001					
22	.533	.847	.513	.159	.020	.001					
23	.519	.844	.503	.150	.018	.001					
24	.505	.840	.494	.143	.016	.001					
25	.492	.837	.485	.136	.015	.001					
26	.479	.834	.477	.130	.013						
27	.468	.831	.470	.124	.012						
28	.456	.829	.463	.119	.011						
29	.446	.826	.456	.114	.010						
30	.435	.824	.450	.110	.009						
60	.256	.783	.355	.056	.003						
∞		.729	.256	.024							

Table I. Table of $E^2_{0.01}$ and the corresponding values of P_{II}

$f_1 = 8$

f_2	$E^2_{.01}$	ϕ									
		1	1.5	2	2.5	3	4	5	6	7	8
2	.997	.979	.965	.946	.923	.894	.826	.746	.659	.569	.481
4	.967	.962	.912	.826	.705	.562	.284	.104	.027	.005	.001
6	.915	.944	.844	.668	.447	.246	.039	.003			
7	.887	.934	.809	.592	.343	.151	.007				
8	.858	.925	.775	.522	.261	.093	.004				
9	.829	.917	.743	.461	.199	.056					
10	.802	.908	.712	.408	.153	.035					
11	.775	.901	.684	.363	.118	.022					
12	.750	.893	.658	.324	.092	.014					
13	.726	.886	.634	.290	.073	.009					
14	.703	.880	.612	.261	.058	.006					
15	.681	.874	.591	.237	.047	.004					
16	.660	.868	.573	.216	.039	.003					
17	.641	.862	.555	.197	.032	.002					
18	.622	.857	.539	.181	.027	.002					
19	.605	.852	.525	.168	.023	.001					
20	.588	.848	.511	.156	.019	.001					
21	.572	.843	.499	.145	.017	.001					
22	.557	.839	.487	.135	.014						
23	.542	.835	.476	.127	.012						
24	.529	.832	.466	.119	.011						
25	.515	.828	.456	.113	.010						
26	.503	.825	.447	.107	.009						
27	.491	.822	.439	.101	.008						
28	.480	.819	.432	.096	.007						
29	.469	.816	.425	.092	.006						
30	.458	.813	.418	.088	.006						
60	.274	.766	.315	.039	.001						
∞		.702	.211	.014							

Tabl. II. Table of $E^2_{0.05}$ and the corresponding values of P_{II}

$f_1 = 1$

f_2	$E^2_{0,05}$	ϕ									
		1	1.5	2	2.5	3	4	5	6	7	8
2	.903	.862	.763	.643	.517	.395	,200	,083	,028	,008	,002
4	.658	.805	.631	.428	,247	,120	.016	,001			
6	.500	.777	.570	.343	.164	,061	,004				
7	.444	.768	.552	.319	.144	.050	,003				
8	.399	.761	.537	.302	.129	,041	,002				
9	.362	.756	.526	,288	.119	,036	,001				
10	.332	.751	.517	.278	.111	,032	,001				
11	.306	.747	.510	.269	.105	,029	,001				
12	.284	.744	.504	.262	.100	,027	,001				
13	.264	.741	.499	.256	,096	.025	,001				
14	.247	.739	.494	,251	.093	,024	,001				
15	.232	.737	.490	.247	,090	,023					
16	.219	.735	.487	.243	,087	,022					
17	.207	.734	.484	.240	.085	,021					
18	.197	.732	.481	.237	,084	,020					
19	.187	.731	.479	.235	,082	,020					
20	.179	.730	.477	,233	,081	,019					
21	.171	.729	.475	.231	,079	,019					
22	.164	.728	.473	,229	,078	,018					
23	.157	.727	.471	.227	.077	.018					
24	.151	.726	.470	.226	.076	,018					
25	.145	.725	.468	.224	,075	,017					
26	.140	.725	.467	.223	,075	,017					
27	.135	.724	.466	.222	.074	,017					
28	.130	.723	.465	.221	,073	,017					
29	.126	.723	.464	.220	,073	,017					
30	.122	.722	.463	.219	,072	,016					
60	.063	.715	,450	,205	,065	,014					
∞		.707	.437	.193	,058	,011					

Table II. Table of $E^2_{0.05}$ and the corresponding values of P_{II}

$f_1 = 2$

f_2	$E^2_{0.05}$	ϕ									
		1	1.5	2	2.5	3	4	5	6	7	8
2	.950	.881	.803	.704	.595	.484	.286	.146	.064	.024	.008
4	.776	.824	.661	.460	.272	.135	.020	.001			
6	.632	.789	.579	.340	.153	.052	.002				
7	.575	.777	.551	.304	.124	.037	.001				
8	.527	.767	.530	.277	.104	.027	.001				
9	.486	.759	.513	.257	.090	.022					
10	.451	.752	.498	.241	.080	.017					
11	.420	.747	.486	.228	.072	.015					
12	.393	.742	.476	.217	.066	.013					
13	.369	.737	.468	.208	.061	.011					
14	.348	.734	.461	.201	.057	.010					
15	.329	.730	.454	.195	.054	.009					
16	.312	.727	.448	.189	.051	.008					
17	.297	.725	.443	.184	.048	.008					
18	.283	.722	.439	.180	.046	.007					
19	.270	.720	.435	.177	.044	.007					
20	.259	.718	.431	.173	.043	.006					
21	.248	.717	.428	.170	.042	.006					
22	.238	.715	.425	.168	.040	.006					
23	.229	.714	.422	.165	.039	.006					
24	.221	.712	.420	.163	.038	.005					
25	.213	.711	.417	.161	.037	.005					
26	.206	.710	.415	.159	.037	.005					
27	.199	.709	.413	.157	.036	.005					
28	.193	.708	.411	.155	.035	.005					
29	.187	.707	.410	.154	.035	.004					
30	.181	.706	.408	.153	.034	.004					
60	.095	.692	.384	.134	.027	.003					
∞		.678	.362	.117	.021	.002					

Table II. Table of $E^2_{0.05}$ and the corresponding values of P_{II}

$f_1 = 3$

f_2	$E^2_{0.05}$	ϕ									
		1	1.5	2	2.5	3	4	5	6	7	8
2	.966	.888	.817	.726	.624	.519	.324	.177	.084	.035	.013
4	.832	.830	.670	.468	.278	.139	.020	.001			
6	.704	.791	.574	.326	.139	.044	.002				
7	.651	.776	.540	.283	.106	.028					
8	.604	.764	.513	.251	.084	.018					
9	.563	.754	.491	.226	.068	.013					
10	.527	.745	.472	.206	.057	.010					
11	.495	.738	.457	.190	.049	.008					
12	.466	.731	.444	.178	.043	.006					
13	.440	.726	.433	.167	.038	.005					
14	.418	.721	.422	.158	.035	.004					
15	.397	.716	.414	.151	.032	.004					
16	.378	.712	.406	.144	.029	.003					
17	.361	.709	.399	.139	.027	.003					
18	.345	.705	.393	.134	.025	.002					
19	.331	.702	.388	.130	.024	.002					
20	.317	.700	.383	.126	.022	.002					
21	.305	.697	.379	.123	.021	.002					
22	.294	.695	.375	.119	.020	.002					
23	.283	.693	.371	.117	.019	.002					
24	.273	.691	.367	.114	.019	.001					
25	.264	.689	.364	.112	.018	.001					
26	.255	.687	.361	.110	.017	.001					
27	.248	.686	.359	.108	.017	.001					
28	.240	.684	.356	.106	.016	.001					
29	.233	.683	.354	.105	.016	.001					
30	.226	.682	.352	.103	.015	.001					
60	.121	.662	.320	.083	.010	.001					
∞		.642	.289	.067	.007						

Table II. Table of $E^2_{0.05}$ and the corresponding values of P_{II}

$f_1 = 4$

f_2	$E^2_{0.05}$	ϕ									
		1	1.5	2	2.5	3	4	5	6	7	8
2	.975	.892	.824	.738	.640	.537	.345	.195	.097	.043	.017
4	.865	.833	.673	.471	.279	.139	.020	.001			
6	.751	.791	.567	.314	.128	.038	.001				
7	.702	.774	.529	.265	.092	.022					
8	.657	.760	.497	.229	.069	.013					
9	.618	.748	.471	.201	.054	.008					
10	.582	.738	.449	.179	.043	.006					
11	.550	.729	.430	.161	.035	.004					
12	.521	.721	.414	.148	.030	.003					
13	.494	.714	.401	.136	.025	.002					
14	.471	.708	.389	.127	.022	.002					
15	.449	.702	.378	.119	.019	.002					
16	.429	.697	.369	.112	.017	.001					
17	.411	.693	.361	.106	.016	.001					
18	.394	.689	.354	.101	.014	.001					
19	.379	.685	.347	.097	.013	.001					
20	.364	.681	.341	.093	.012	.001					
21	.351	.678	.335	.089	.011	.001					
22	.339	.675	.331	.086	.010	.001					
23	.327	.672	.326	.083	.010						
24	.316	.670	.322	.080	.009						
25	.306	.668	.318	.078	.009						
26	.297	.665	.315	.076	.008						
27	.288	.663	.312	.074	.008						
28	.279	.661	.309	.072	.008						
29	.272	.660	.306	.071	.007						
30	.264	.658	.303	.069	.007						
60	.144	.632	.265	.049	.004						
∞		.604	.227	.036	.002						

Table II. Table of $E^2_{0.05}$ and the corresponding values of P_{II}

$f_1 = 5$

f_2	$E^2_{0.05}$	ϕ									
		1	1.5	2	2.5	3	4	5	6	7	8
2	.980	.894	.828	.745	.649	.549	.359	.207	.106	.043	.019
4	.887	.835	.675	.473	.280	.138	.020	.001			
6	.785	.790	.561	.304	.119	.033	.001				
7	.739	.772	.519	.251	.082	.018					
8	.697	.756	.483	.211	.059	.010					
9	.659	.743	.454	.181	.044	.006					
10	.625	.731	.429	.158	.033	.004					
11	.593	.720	.408	.140	.026	.002					
12	.564	.711	.390	.125	.021	.002					
13	.538	.703	.374	.113	.017	.001					
14	.514	.695	.360	.103	.015	.001					
15	.492	.689	.348	.095	.012	.001					
16	.471	.683	.338	.088	.011	.001					
17	.452	.678	.328	.083	.009						
18	.435	.673	.320	.078	.008						
19	.419	.668	.312	.073	.007						
20	.404	.664	.305	.069	.007						
21	.390	.660	.299	.066	.006						
22	.377	.656	.294	.063	.006						
23	.365	.653	.288	.060	.005						
24	.353	.650	.284	.058	.005						
25	.342	.647	.279	.056	.005						
26	.332	.644	.275	.054	.004						
27	.323	.642	.272	.052	.004						
28	.314	.640	.268	.050	.004						
29	.305	.637	.265	.049	.003						
30	.297	.635	.262	.048	.003						
60	.165	.604	.219	.031	.001						
∞		.567	.177	.019	.001						

Table II. Table of $E^2_{0.05}$ and the corresponding values of P_{II}

$f_1 = 6$

f_2	$E^2_{0.05}$	ϕ								
		1	1.5	2	2.5	3	4	5	6	7	8
2	.983	.895	.831	.749	.656	.557	.368	.216	.112	.052	.022
4	.902	.836	.677	.473	.280	.138	.019	.001			
6	.811	.789	.556	.296	.113	.030	.001				
7	.768	.769	.510	.239	.074	.015					
8	.729	.753	.472	.198	.051	.008					
9	.692	.738	.440	.166	.037	.005					
10	.659	.725	.412	.142	.027	.003					
11	.628	.713	.389	.123	.020	.002					
12	.600	.702	.369	.108	.016	.001					
13	.574	.693	.351	.096	.012	.001					
14	.550	.685	.336	.086	.010	.001					
15	.527	.677	.323	.078	.008						
16	.507	.669	.311	.071	.007						
17	.488	.663	.301	.065	.006						
18	.470	.657	.291	.061	.005						
19	.454	.652	.283	.056	.004						
20	.438	.648	.276	.053	.004						
21	.424	.644	.269	.050	.003						
22	.410	.639	.262	.047	.003						
23	.397	.635	.257	.045	.003						
24	.385	.632	.252	.043	.003						
25	.374	.629	.247	.041	.002						
26	.363	.625	.242	.039	.002						
27	.353	.623	.238	.037	.002						
28	.344	.620	.234	.036	.002						
29	.335	.617	.231	.034	.002						
30	.326	.615	.228	.033	.002						
60	.184	.576	.181	.019	.001						
∞		.532	.138	.010							

Table II. Table of $E^2_{0.05}$ and the corresponding values of P_{II}

$f_1 = 7$

f_2	$E^2_{0.05}$	ϕ									
		1	1.5	2	2.5	3	4	5	6	7	8
2	.986	.896	.833	.753	.660	.563	.374	.222	.117	.055	.023
4	.914	.837	.678	.474	.280	.138	.019	.001			
6	.831	.788	.552	.289	.108	.028	.001				
7	.791	.767	.503	.230	.068	.013					
8	.754	.749	.462	.187	.046	.007					
9	.719	.733	.427	.154	.031	.004					
10	.687	.719	.398	.129	.022	.002					
11	.657	.706	.373	.110	.016	.001					
12	.630	.695	.351	.094	.012	.001					
13	.604	.684	.332	.082	.009						
14	.580	.675	.316	.073	.007						
15	.558	.667	.301	.065	.006						
16	.538	.659	.289	.058	.005						
17	.518	.652	.277	.053	.004						
18	.501	.645	.267	.048	.003						
19	.484	.639	.258	.044	.003						
20	.468	.634	.250	.041	.002						
21	.453	.629	.243	.038	.002						
22	.439	.624	.236	.036	.002						
23	.426	.619	.230	.034	.002						
24	.414	.615	.224	.032	.001						
25	.402	.611	.219	.030	.001						
26	.391	.607	.215	.028	.001						
27	.381	.604	.210	.027	.001						
28	.371	.601	.206	.026	.001						
29	.362	.598	.202	.024	.001						
30	.353	.595	.199	.023	.001						
60	.202	.550	.150	.012							
∞		.498	.105	.005							

Table II. Table of $E^2_{0.05}$ and the corresponding values of P_{II}

$f_1 = 8$

f_2	$E^2_{0.05}$	ϕ 1	1.5	2	2.5	3	4	5	6	7	8
2	.987	.897	.835	.755	.664	.567	.380	.229	.121	.057	.024
4	.924	.838	.678	.474	.279	.137	.019	.001			
6	.847	.787	.548	.284	.103	.026	.001				
7	.810	.765	.497	.222	.064	.012					
8	.775	.746	.454	.178	.041	.006					
9	.742	.729	.417	.144	.028	.003					
10	.711	.714	.386	.119	.019	.001					
11	.682	.700	.359	.099	.013	.001					
12	.655	.686	.336	.084	.009						
13	.630	.677	.316	.072	.007						
14	.607	.666	.298	.062	.005						
15	.585	.657	.283	.055	.004						
16	.564	.648	.269	.048	.003						
17	.545	.641	.257	.043	.003						
18	.527	.634	.247	.039	.002						
19	.510	.627	.237	.035	.002						
20	.495	.620	.228	.032	.001						
21	.480	.615	.220	.030	.001						
22	.466	.609	.213	.027	.001						
23	.452	.604	.207	.025	.001						
24	.440	.600	.201	.024	.001						
25	.428	.595	.196	.022	.001						
26	.417	.591	.191	.021	.001						
27	.406	.588	.186	.020	.001						
28	.396	.584	.182	.019							
29	.386	.581	.178	.018							
30	.377	.578	.175	.017							
60	.219	.527	.125	.008							
∞		.466	.081	.003							