

Mean concentration function + Quasi mean
concentration function. III

所員 國澤清典

第二章 Quasi mean concentration function.

§ 2.1. Quasi mean concentration function.

(1.1.5) $|f(t)|^2 \in f(t)$, real part $\mathcal{R}(f(t))$

示オキカナルト $F(x)$, quasi mean concentration

function (q. m. c. f.) ヲ得ル:

$$\begin{aligned}\phi_F(h) &= h \int_0^\infty e^{-ht} \mathcal{R}(f(t)) dt \\ &= \int_{-\infty}^\infty \frac{h^2}{h^2 + x^2} dF(x), \quad h > 0.\end{aligned}$$

此, $\phi_F(h)$ 明カニ h , non-negative, non-decreasing & continuous function 示アル. 明カニ

$$\lim_{h \rightarrow \infty} \phi_F(h) = 1.$$

サテ次, lemma ヲ考へル. 此レハ Lemma 1.1.1 と同様ニシテ証明サレル.

Lemma 2.1.1. 如何ナル $T > 0$ ニ對シテモ

$$(2.1.3) \quad 1 - \phi_F(h) \leq C(T) \int_0^T (1 - \mathcal{R}(f(\frac{t}{h}))) dt$$

此 $C = C(T)$ ハ T へニ關係スル const. 示アル.

Theorem 2.1.1. T ンテ $h > 0$ ニ對シテモ

$$(2.1.4) \quad 1 - \phi_F(h) \geq \frac{1}{2} (1 - \psi_F(h))$$

(137)

$\exists 0 < t < T = \text{対シ } Q(f(t)) > \delta > 0 \text{ ナル様}$
 $T \text{ が存在スルナラバ}$

$$(2.1.5) \quad 1 - \Phi_F^2(R) \geq R(T, R) (1 - \Phi_F(R))$$

此処 $= R(T, R)$ の T と R の \equiv 関係スル
 ナルヲ。

証明。 明らか =

$$\begin{aligned}
 1 - |f(t)|^2 &= 1 - (Q(f(t)))^2 - (J(f(t)))^2 \\
 &\leq 2(1 - Q(f(t)))
 \end{aligned}$$

示アルカテ (2.1.4) = 明らか。

假定ヨリ $Q(f(t)) > \delta > 0 \quad (0 \leq t \leq T)$ ナ

$\exists 0 \leq t \leq T = \text{対シ}$

$$\begin{aligned}
 1 - |f(t)|^2 &\geq 1 - Q(f(t)) - (J(f(t)))^2 \\
 &\geq (1 - Q(f(t))) - \int_{-\infty}^{\infty} \sin^2 tx \, dF(x) \\
 &= 1 - Q(f(t)) - \frac{1}{2} \int_{-\infty}^{\infty} (1 - \cos 2tx) \\
 &= 1 - Q(f(t)) - \frac{1}{2} (1 - Q(f(2t)))
 \end{aligned}$$

故 =

$$\begin{aligned}
 &R \int_0^T e^{-Rt} (1 - |f(t)|^2) \, dt \\
 &\geq R \int_0^T e^{-Rt} (1 - Q(f(t))) \, dt - \frac{R}{2} \int_0^T e^{-Rt} \\
 &\quad Q(f(2t)) \, dt \geq \\
 &\geq \int_0^{2T} (e^{-t} - \frac{1}{2} e^{-\frac{t}{2}}) (1 - Q(f(\frac{t}{2}))) \, dt
 \end{aligned}$$

一方十分小ナル $\eta > 0 =$ 対シ $T_0 > 0$ ヲ選ビ

$0 \leq t \leq T_0 =$ 対シ

$$e^{-t} - \frac{1}{4} e^{-\frac{t}{2}} \geq \eta > 0$$

従ツテ $T_1 = \min. \{T_0, R_T\} =$ 対シ

$$1 - \psi_F(R) \geq R \int_0^T e^{-Rt} (1 - |f(t)|^2) dt$$

$$\geq \int_0^{RT} (e^{-t} - \frac{1}{4} e^{-\frac{t}{2}}) (1 - Q(t/\frac{T}{R})) dt$$

$$\geq \eta \int_0^{T_1} (1 - Q(f(t))) dt$$

Lemma 2.1.1 $\exists \eta$

$$1 - \psi_F(R) \geq \eta \int_0^{T_1} (1 - Q(f(t))) dt$$

$$\geq \frac{\eta}{C(T)} (1 - \rho_F(R))$$

$$\eta/C(T) = R(T, R) \text{ トオケバ}$$

$$1 - \psi_F(R) \geq R(T, R) (1 - \rho_F(R))$$

定理 2.1.2. $\{F_1(x), \dots, F_n(x)\}$ \rightarrow 有限

数, 任意, 集合トスル

$$f_R(t) \equiv \int_{-\infty}^{\infty} e^{itx} dF_R(x) \equiv \int_{-\infty}^{\infty} e^{itx} dF_n(x + \int_{-A}^A x dF_n(x))$$

($A > 0$)

トオケトラバ

$$\sum_{k=1}^n |f_k(t/A) - 1| \leq (t^2 + 2|t| + 4) \sum_{k=1}^n (1 - \rho_k(A))$$

証明

□ (given)

(Handwritten notes and scribbles at the bottom of the page)

簡單，夕×

(139)

$$b_k = \int_{-A}^A x dF_k(x), \quad (k=1, 2, \dots, n)$$

$$\sum_{k=1}^n \left| f_k' \left(\frac{t}{A} \right) - 1 \right| = \sum_{k=1}^n \left| \int_{-\infty}^{\infty} e^{\frac{itx}{A}} dF_k'(x) \right|$$

$$= \sum_{k=1}^n \left| \int_{-\infty}^{\infty} \left(e^{it \frac{(x-b_k)}{A}} - 1 \right) dF_k(x) \right|$$

$$\leq \sum_{k=1}^n \left\{ \int_{|x| \leq A} \left(e^{it \frac{(x-b_k)}{A}} - 1 \right) dF_k(x) \right\} +$$

$$\leq \sum_{k=1}^n \left\{ \int_{|x| \leq A} \left(e^{it \frac{(x-b_k)}{A}} - \frac{it(x-b_k)}{A} - 1 \right) dF_k(x) \right.$$

$$\left. + \sum_{k=1}^n |t| \int_{|x| \leq A} \frac{x-b_k}{A} dF_k(x) \right\} + 2 \sum_{k=1}^n \int_{|x| > A}$$

$$\leq \sum_{k=1}^n \left\{ \frac{t^2}{2} \int_{|x| \leq A} \frac{(x-b_k)^2}{A^2} dF_k(x) + \sum_{k=1}^n |t| \frac{|b_k|}{A} \right.$$

$$\left. + 2 \sum_{k=1}^n \int_{|x| > A} dF_k(x) \right\}$$

$$\leq \sum_{k=1}^n \left\{ \frac{t^2}{2} \left(\int_{|x| \leq A} \frac{x^2}{A^2} dF_k(x) - \frac{2b_k^2}{A^2} + \frac{b_k^2}{A^2} \right) \right.$$

$$\left. + |t| \int_{|x| > A} dF_k(x) + 2 \int_{|x| > A} dF_k(x) \right\}$$

$$\leq \sum_{k=1}^n \left\{ \frac{t^2}{2} \int_{|x| \leq A} \frac{x^2}{A^2} dF_k(x) + |t| \int_{|x| > A} dF_k(x) \right.$$

$$\left. + 2 \int_{|x| > A} dF_k(x) \right\} \leq (t^2 + 2|t| + 4) \sum_{k=1}^n \int_{|x| > A} dF_k(x)$$

$$= (t^2 + 2|t| + 4) \sum_{k=1}^n (1 - \phi_k(A))$$

以上, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000

§ 2.2. The relation between g. m. c. f.

and the convolution of probability distribution.

g. m. c. f. の性質に属する probability distribution, convolution. = 3, 1, 減少に + 1. 例

例

$$F(x) = \begin{cases} 0 & x < -1 \\ \frac{1}{2} & -1 \leq x < 1 \\ 1 & 1 \leq x \end{cases}$$

を考へて $f(t) = \int_{-\infty}^{\infty} e^{itx} dF(x) = \cos t$

故に

$$\phi_F(h) = h \int_0^{\infty} e^{-ht} \mathcal{R}(f(t)) dt = \frac{h^2}{h^2 + 1}$$

一方

$$\begin{aligned} \phi_{F*F}(h) &= h \int_0^{\infty} e^{-ht} \mathcal{R}(f^2(t)) dt \\ &= h \int_0^{\infty} e^{-ht} \left(\frac{\cos 2t + 1}{2} \right) dt \\ &= \frac{1}{2} \left(\frac{h^2}{h^2 + 4} + 1 \right) \end{aligned}$$

故に $h < \sqrt{2}$ に対し

$$\phi_F(h) < \phi_{F*F}(h)$$

然し定理 1.2.1 の中、相似定理を示

不事が出来ル。

random variables, system (1.2.1) $\parallel X_{nm} \parallel$
が與テラレ。 $F_{nm}(x)$ $\rightarrow X_{nm}$, 分布函数トスル
 $D_n(\alpha)$ \rightarrow 次ノ様ニシテ定義サレル函数ト
スル

$$(2.2.1) \quad \alpha = \phi_{F_{n_1}} * \dots * \phi_{F_{n_{m_n}}} (D_n(\alpha)), \quad (1 > \alpha > 0)$$

定理 2.2.1. 二ヶノ実数 α, β ($\frac{3}{4} < \alpha \leq 1, \frac{7}{8} < \beta \leq 1$) が与テラレ時 次ノ関係
ヲミテ正ノ const. K ト N (共 $= \alpha + \beta$)
ニ Depend スル) \rightarrow 定メラルテが出来ル。即
チ $m_n > N, D_n(\beta) \geq l_0$ 且 \forall

$$(2.2.2) \quad \phi_{F_{n_m}}(l_0) = \alpha, \quad m = 1, 2, \dots, m_n$$

がミテサレテラレ

$$(2.2.3) \quad D_n(\beta) \geq \sqrt{m_n} l_0 K, \quad (m_n \geq N)$$

從 \forall \rightarrow

$$(2.2.4) \quad \phi_{F_{n_1}} * \dots * \phi_{F_{n_{m_n}}} (\sqrt{m_n} l_0 K) \leq \beta.$$

証明 定理 2.1.1, (2.1.4) \rightarrow \forall

$$(2.2.5) \quad \begin{aligned} 1 - \beta &= \int_{-\infty}^{\infty} \frac{x^2}{D_n^2(\beta) + x^2} dF_{n_1} * \dots * F_{n_{m_n}}(x) \\ &\geq \frac{1}{2} \int_{-\infty}^{\infty} \frac{x^2}{D_n^2(\beta) + x^2} d\tilde{F}_{n_1} * \dots * \tilde{F}_{n_{m_n}}(x) \end{aligned}$$

$$\geq \frac{1}{4} \left\{ \int_{|x| \leq D_n(\beta)} \frac{x^2}{D_n^2(\beta)} d\tilde{F}_{n1} * \dots * \tilde{F}_{nm_n}(x) \right. \\ \left. + \int_{|x| > D_n(\beta)} d\tilde{F}_{n1} * \dots * \tilde{F}_{nm_n}(x) \right.$$

- 5

$$1 - \prod_{m=1}^{m_n} \left| t_{nm} \left(\frac{t}{D_n(\beta)} \right) \right|^2 = \int_{-\infty}^{\infty} \left(1 - \cos \frac{tx}{D_n(\beta)} \right) d\tilde{F}_{n1} * \dots * \tilde{F}_{nm_n}(x) \\ = 2 \int_{-\infty}^{\infty} \sin^2 \left(\frac{tx}{2D_n(\beta)} \right) d\tilde{F}_{n1} * \dots * \tilde{F}_{nm_n}(x) \leq \\ \leq 2 \left\{ t^2 \int_{|x| \leq D_n(\beta)} \frac{x^2}{4D_n^2(\beta)} d\tilde{F}_{n1} * \dots * \tilde{F}_{nm_n}(x) \right. \\ \left. + \int_{|x| > D_n(\beta)} d\tilde{F}_{n1} * \dots * \tilde{F}_{nm_n}(x) \right.$$

$12\beta > \frac{\sqrt{3}}{8} t$ (2.2.5) \exists $0 \leq t \leq 2 = \bar{t}$ 対シ

$$1 - \prod_{m=1}^{m_n} \left| t_{nm} \left(\frac{t}{D_n(\beta)} \right) \right|^2 \leq 2 \left\{ \int_{|x| \leq D_n(\beta)} \frac{x^2}{D_n^2(\beta)} d\tilde{F}_{n1} * \dots * \tilde{F}_{nm_n}(x) \right. \\ \left. + \int_{|x| > D_n(\beta)} d\tilde{F}_{n1} * \dots * \tilde{F}_{nm_n}(x) \right\} \leq \delta(1-\beta) = \delta' < 1$$

故 $0 \leq t \leq 2 = \bar{t}$ 対シ

$$0 < \delta = 1 - \delta' \leq \prod_{m=1}^{m_n} \left| t_{nm} \left(\frac{t}{D_n(\beta)} \right) \right|^2 = \\ = \prod_{m=1}^{m_n} \left| \int_{-\infty}^{\infty} e^{itx} d\tilde{F}(D_n(\beta)x) \right|^2$$

定理 1.1.2 \exists

$$\sum_{m=1}^{m_n} (1 - \psi_{G_{nm}}) \leq K (1 - \psi_{G_{n1} * \dots * G_{nm}}) \quad (1)$$

但し K は abs. const., $G_{nm}(x) \equiv F_{nm}(D_n(\beta)x)$

(2.2.5) より

$$\begin{aligned} \sum_{m=1}^{m_n} (1 - \psi_{G_{nm}}(1)) &= \sum_{m=1}^{m_n} \int_0^{\infty} e^{-t} (1 - |f_{nm}(\frac{t}{D_n(\beta)})|^2) dt \\ &= \sum_{m=1}^{m_n} \int_{-\infty}^{\infty} \frac{x^2}{D_n^2(\beta) + x^2} d\tilde{F}_{nm}(x) \leq K (1 - \psi_{G_{n1}}(1) \cdots \psi_{G_{nm_n}}(1)) \\ &= K \int_0^{\infty} e^{-t} (1 - \prod_{m=1}^{m_n} |f_{nm}(\frac{t}{D_n(\beta)})|^2) dt \leq 2(1-\beta)K \end{aligned}$$

他方 (2.2.2) より $D_n(\beta) \geq l_0 \exists$

$$\begin{aligned} (2.2.7) \quad 1 - \alpha &= \int_{-\infty}^{\infty} \frac{x^2}{l_0^2 + x^2} dF_{nm}(x) \geq \int_{-\infty}^{\infty} \frac{x^2}{D_n^2(\beta) + x^2} \\ &\geq \frac{1}{2} \left\{ \int_{|x| \leq D_n(\beta)} \frac{x^2}{D_n^2(\beta)} dF_{nm}(x) + \int_{|x| > D_n(\beta)} dF_{nm}(x) \right\}, \end{aligned}$$

$m = 1, 2, \dots, m_n$

然し

$$\begin{aligned} 1 - \mathcal{Q}(f_{nm}(\frac{t}{D_n(\beta)})) &= \int (1 - \cos \frac{tx}{D_n(\beta)}) dF_{nm}(x) \\ &= 2 \int_{-\infty}^{\infty} \frac{\sin^2 \frac{tx}{2D_n(\beta)}}{2D_n(\beta)} dF_{nm}(x) \\ &\leq 2 \left\{ t^2 \int_{|x| \leq D_n(\beta)} \frac{x^2}{4D_n^2(\beta)} dF_{nm}(x) + \int_{|x| > D_n(\beta)} dF_{nm}(x) \right\}, \end{aligned}$$

$m = 1, 2, \dots, m_n$

従って (2.2.7) より $0 \leq t \leq 2$ に対し

$$1 - \mathcal{Q}(f_{nm}(\frac{t}{D_n(\beta)})) \leq 4(1 - \alpha),$$

$m = 1, 2, \dots, m_n$

故 = $3/4 < \alpha \leq 1$ かつ $0 \leq t \leq 2 =$ 対し

$$0 < 1 - 4(1 - \alpha) \leq R (f_{nm}(\frac{t}{D_n(\beta)})),$$

$$m = 1, 2, \dots, m_n$$

定理 2.1.1, (2.1.5) より

$$1 - \psi_{G_{nm}}(t) \geq R(1 - \phi_{G_{nm}}(t)) = R \int_{-\infty}^{\infty} \frac{x^2}{D_n^2(\beta) + x^2} dF_{nm}(x)$$

$$m = 1, 2, \dots, m_n$$

但し R は abo. const. (2.2.6) より

$$\frac{2(1-\beta)K}{R} \geq \sum_{m=1}^{m_n} \int_{-\infty}^{\infty} \frac{x^2}{D_n^2(\beta) + x^2} dF_{nm}(x)$$

$$\geq \sum_{m=1}^{m_n} \frac{l_0^2}{D_n^2(\beta) + l_0^2} \left\{ \int_{|x| \leq l_0} x^2/l_0^2 dF_{nm}(x) + \int_{|x| > l_0} \alpha dF_{nm}(x) \right\} \geq \sum_{m=1}^{m_n} \frac{l_0^2}{D_n^2(\beta) + l_0^2} \int_{-\infty}^{\infty} \frac{x^2}{l_0^2 + x^2} dF_{nm}(x)$$

(2.2.2) より

$$\frac{2(1-\beta)K}{R} \geq \sum_{m=1}^{m_n} \frac{l_0^2}{D_n^2(\beta) + l_0^2} \int_{-\infty}^{\infty} \frac{x^2}{l_0^2 + x^2} dF_{nm}(x)$$

$$= \frac{m_n l_0^2}{D_n^2(\beta) + l_0^2} (1 - \alpha)$$

故に

$$D_n(\beta) \geq \sqrt{m_n} l_0 \sqrt{\frac{(1-\alpha)R}{2(1-\beta)K} - \frac{1}{m_n}}$$

$$\text{依りて } m_n > N(\alpha, \beta) = \frac{2(1-\beta)K}{(1-\alpha)R} + \epsilon^{-1}$$

$$D_n(\beta) \geq \sqrt{m_n} l_0 K, \quad K = \sqrt{\frac{(1-\alpha)R}{2(1-\beta)K} - 1}$$

(145)

故 =

$$\phi_{F_{m_1}^* \dots * F_{m_n}^*}(\sqrt{m_n} \theta_0, K) \leq \beta$$

但 $\Rightarrow K$ 与 α, β 有关 \equiv depend on

" "