Information transmission using non-Poisson regular firing

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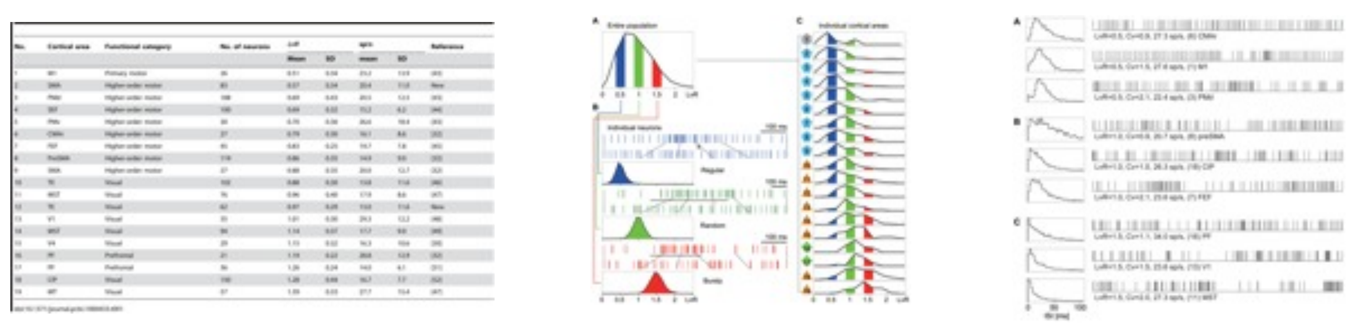
Abstract

In many cortical areas neural spike trains are non-Poisson (Shinomoto et al., 2009). Here, we investigate a possible benefit of non-Poisson spiking for information transmission by studying the minimal rate fluctuation that can be detected by a downstream optimal observer, i.e., a Bayesian estimator. The idea is that an inhomogeneous Poisson process may make it difficult for downstream decoders to resolve subtle changes in rate fluctuation, but by using a more regular non-Poisson process the nervous system can make rate fluctuations easier to detect and, therefore, more informative. We evaluate the degree to which regular firing reduces the rate fluctuation detection threshold. We find that the threshold for detection is reduced as the coefficient of variation of interspike intervals increases.

Motivation

- Neurons in cortical areas exhibit stable firing patterns that can be characterized in terms of "local variation (LV)" of interspike intervals(ISIs).
- There is a strong correlation between the type of signaling pattern exhibited by neurons in a given area and the function of that area.

$$LvR = \frac{3}{n-1}\sum_{i=1}^{n-1}\left(1-\frac{4I_iI_{i+1}}{(I_i+I_{i+1})^2}\right)\left(1+\frac{4R}{I_i+I_{i+1}}\right), \quad I_i: i\text{-th ISIs.}$$



Shinomoto et al., PLoS Computational Biology (2009) 5:e1000433.

Inference problem

Goal:
Given a sequence of events $\{t_1, t_2, ..., t_n\}$, infer the firing rate $\lambda(t)$ and regularity κ.

Construction of the generative model

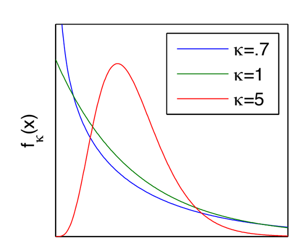
i) ISI distribution: gamma distribution

$$f_\kappa(x) = \kappa(\kappa x)^{\kappa-1}e^{-\kappa x}/\Gamma(\kappa)$$

where

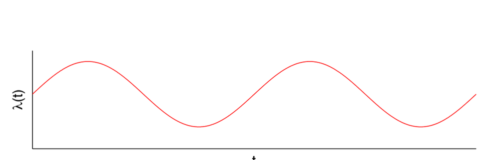
$$\int_0^\infty x f_\kappa(x)dx = 1$$

$$\kappa = 1/C_V^2 = E(X)^2/Var(X)$$



ii) Firing rate

$$\lambda(t) = E\left[\sum_i \delta(t-t_i)\right]$$

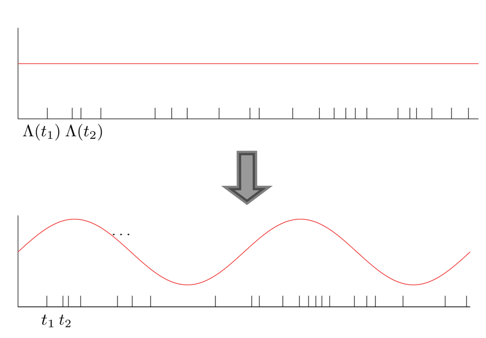


By applying the time-rescaling transformation,

$$\Lambda(t_i) = \int_0^{t_i}\lambda(t)dt,$$

the probability density of $\{t_1, t_2, ..., t_n\}$ is obtained as

$$p_\kappa(\{t_i\}|\{\lambda(t)\}) = \prod_i \lambda(t_i)f_\kappa(\Lambda(t_i)-\Lambda(t_{i-1})).$$



Empirical Bayes decoding

The inverse probability of the firing rate $\lambda(t)$ given the spike train $\{t_1, t_2, ..., t_n\}$ is obtained according to the Bayes' rule

$$p_{\kappa,\gamma}(\{\lambda(t)\}|\{t_i\}) = \frac{p_\kappa(\{t_i\}|\{\lambda(t)\})p_\gamma(\{\lambda(t)\})}{p_{\kappa,\gamma}(\{t_i\})}.$$

We choose the prior distribution of the firing rate such that the large gradient of $\lambda(t)$ is penalized with

$$p_\gamma(\{\lambda(t)\}) = \frac{1}{Z(\gamma)}\exp\left(-\frac{1}{2\gamma^2}\int_0^T(d\lambda/dt)^2dt\right),$$

where the hyperparameter γ controls the flatness of the time-dependent rate $\lambda(t)$; with small value of γ, the model requires the flat firing rate and vice versa.

The optimal hyperparameters γ and κ are determined by maximizing the marginal likelihood defined by

$$p_{\kappa,\gamma}(\{t_i\}) = \int D\{\lambda(t)\}p_\kappa(\{t_i\}|\{\lambda(t)\})p_\gamma(\{\lambda(t)\}).$$

The path integral method

Path integral representation of the marginal likelihood

By decomposing the firing rate into the mean μ and fluctuation $x(t)$,

$$\lambda(t) = \mu + x(t)$$

the marginal likelihood can be factorized as

$$p_{\kappa,\gamma}(\{t_i\}) = e^{\mathcal{L}(\kappa)}\cdot\mathcal{F}(\kappa,\gamma)$$

where

$$e^{\mathcal{L}(\kappa)} = \prod_i Gamma(t_i - t_{i-1}|\mu,\kappa) \quad \text{: likelihood of the gamma distribution}$$

$$\mathcal{F}(\kappa,\gamma) = \frac{1}{Z(\gamma)}\int D\{x(t)\}\exp\left[-\int_0^T L(\dot{x},x)dt\right] \quad \text{: contribution of the rate fluctuation}$$

with the Lagrangian

$$L(\dot{x},x) = \frac{1}{2\gamma^2}\dot{x}^2 + \kappa x(t) - \kappa\sum_i\delta(t-t_i)\log\left(1+\frac{x(t)}{\mu}\right)$$

Semiclassical approximation (infinite dimensional Laplace approximation)

Let $\hat{x}(t)$ be the MAP path that satisfies the Euler-Lagrange equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\hat{x}}}\right) - \frac{\partial L}{\partial\hat{x}} = 0$$

By approximating the action integral to the range of quadratic, the path integral can be performed analytically as

$$\mathcal{F}(\kappa,\gamma) = \frac{R}{Z(\gamma)}\exp\left[-\int_0^T L(\dot{\hat{x}},\hat{x})dt\right]$$

where R represents the "quantum" contribution of the quadratic derivation to the path integral:

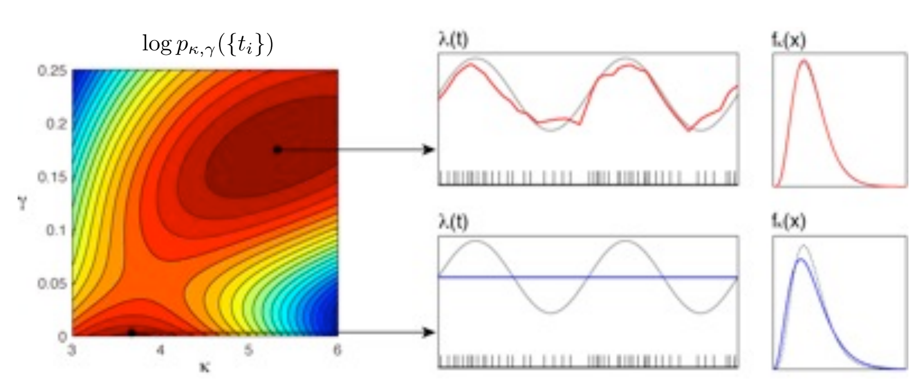
$$R = \frac{1}{\sqrt{2\pi\gamma^2T}}\left[\frac{\det\left(-\partial_t^2+\gamma^2\frac{\partial^2L}{\partial x^2}\right)}{\det\left(-\partial_t^2\right)}\right]^{-\frac{1}{2}}$$

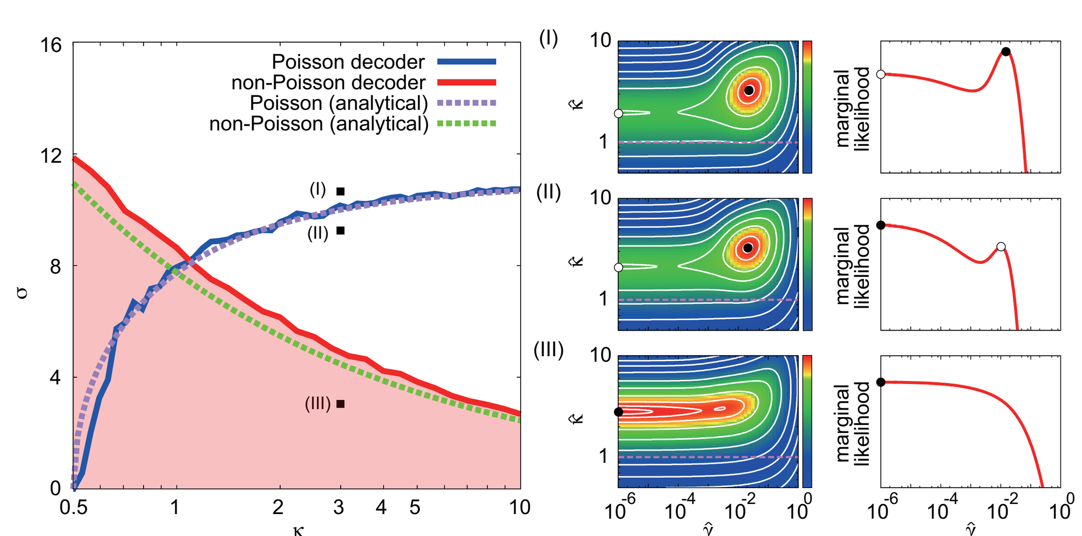
The ratio of determinants can be computed in the standard way (i.e., Gelfand-Yaglom formula).

Results

Example 1 : Sinusoidally modulated rate $\lambda(t) = \mu + \sigma\sin t/\tau$

Two interpretations for a given spike sequence





First-order phase transition　　Critical points : $\kappa\sigma^2\tau/\mu = 2$

Example 2 : OUP modulated rate

$$\frac{d\lambda}{dt} = -\frac{\lambda-\mu}{\tau} + \sigma\sqrt{\frac{2}{\tau}}\xi(t)$$

Second-order phase transition

Critical points : $\kappa\sigma^2\tau/\mu = 1/2$