On parameter estimation based on the contact distance for certain superposed Neyman-Scott spatial cluster fields

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1　Abstract

In Tanaka(2009), we consider the parameter estimation of the superposed Neyman-Scott spatial cluster field, which is a typical cluster point pattern model. We show that the parameters of this model whose *J*-functions satisfy certain condition can be estimated via an isotropic and inhomogeneous Poisson maximum likelihood analysis based on contact distance, which is defined as the shortest distance from the given location to the nearest point. A sufficient condition for our method to hold is that the maximum log-likelihood function is unimodal. Our proposed method makes it feasible to generalize the number of superposition as well.

2　Preliminaries

A Neyman-Scott spatial cluster field is a collection of homogeneous, isotropic and independent clusters, whose centres form a homogeneous Poisson field with intensity μ. Neyman and Scott originally proposed the model to describe astronomical galaxies. An observed window for cluster fields is prescribed to be a unit square, which is denoted by A. We assume that cluster fields are endowed with homogeneity and isotropy, as well as a periodic boundary condition.

Definition. The Neyman-Scott spatial cluster field is constructed as follows: the unobserved ***cluster centres*** are generated via a homogeneous Poisson field with intensity μ. Each individual cluster centre produces a random number M of ***cluster points***, which are realized independently and identically. The distribution of M is given by the $\Pr\{M = m\}$, $m = 0, 1, \ldots$. The corresponding mean is denoted by ν. The cluster points are isotropically scattered around the cluster centres, furthermore the distances between each individual cluster centre and its points are independent and identically distributed according to the dispersive kernel q_τ. The resulting from a collection of such all cluster points, except for each individual cluster centre, is called the ***Neyman-Scott spatial cluster field***.

Definition. The notion of Neyman-Scott spatial cluster field can be trivially generalized to ***superposed Neyman-Scott spatial cluster field,*** X, as a collection of Neyman-Scott spatial cluster field, X_i, with different cluster sizes, denoted by the parameter set (μ_i, ν_i, τ_i).

Definition. Let X be a point field in A. For any point u in A, we denote by $dist(u,X)$ the shortest distance from the given location u to the nearest point of X. This distance is known as the ***contact distance***.

Definition. The ***spherical contact distance function*** F is the cumulative distribution of the contact distance $dist(u,X)$. The ***nearest neighbor distance function*** G is the cumulative distribution of the contact distance $dist(x, X\setminus\{x\})$ for any point x in X.

Definition. (*J*-function) For any $r \geq 0$ for which $F(r) < 1$, $J(r) := (1\text{-}G(r))/(1\text{-}F(r))$.

3 Main results

We assume that each individual derivative of the nearest neighbor distance function of X_i, $i = 1,..., k$, to be positive. Let a_i be each individual ratio of an intensity of X_i to a total one of X, $i = 1,..., k$. We further set $a = (a_1,...,a_k)$. We also assume that each g_a is regarded as a inhomogeneous Poisson intensity with rotation invariant. The ***isotropic and inhomogeneous Poisson maximum log-likelihood function*** $log\ L$ based on the ***NND intensity function*** (derivative of the nearest neighbor distance function) with respect to a is of the following form (Tanaka and Ogata(2009)): $\boldsymbol{log\ L(a) = \sum_j log\ g_a(r_j) - G_a(1/2)}$, $j = 1,..., N,$ where N denotes the total number of location data and r_j denotes the contact distance of each individual location in the data set. We obtain a sufficient condition for $log\ L$ to be unimodal as follows:

Theorem. Suppose that, for $i = 1,..., k$, $F_i < 1$, $J_i(r_j)$ does not coincide with $(1\text{-}G_i(r))/(1\text{-} F(r_j))$, where each G_i is locally constant on sufficiently small neighborhood of some contact distance r_j, $j \leq N$. Then $log\ L$ admits itself to be unimodal at its critical point.

Theorem. Suppose that $log\ L$ admits itself to be unimodal with respect to each individual component of a. Then each individual parameter set $(\boldsymbol{\mu_i, \nu_i})$ of X is completely determined.

References

Tanaka, U. (2009). On parameter estimation based on the contact distance for certain superposed Neyman-Scott spatial cluster fields, Res. Memo, No.1104, (submitted).

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