ヒルベルト空間におけるマルチンゲール中心極限定理とその応用

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Goodness of fit tests play an important role in theoretical and applied statistics, and the study for them has a long history. Such tests are really useful especially if they are *distribution free*, in the sense that their distributions do not depend on the underlying model. The origin goes back to the Kolmogorov-Smirnov and Crámer-von Mises tests in the i.i.d. case, established early in the 20th century, which are *asymptotically distribution free*.

Key tools to derive the asymptotic distribution results for those statistics are the *invariance principles*, that is, weak convergence theorems in function spaces. As for the Kolmogorov-Smirnov type tests, a right choice for the function space is $\ell^\infty(T)$, the space of bounded functions on a set T with the uniform metric, or the space of càdlàg functions with the Skorohod topology. However, as for the Crámer-von Mises type tests it is more natural to treat them as $L_2(\nu)$-valued random elements where ν is a finite measure. Since Hoffman-Jorgensen and Pisier (1976) it has been known that separable Hilbert spaces are the only infinite-dimensional Banach spaces for which the classical central limit property for i.i.d. sequences is equivalent to the square integrability of the norm of the variable. So we choose separable Hilbert spaces as the framework for our general argument on functional central limit theorems. The first goal in this paper is to present such a theorem in terms of martingale type assumptions. Dedecker and Merlevède (2003) and Merlevède (2003) gave some results based on certain mixing conditions. The point of our work is to avoid those conditions keeping some applications for ergodic diffusions stated below in mind. Although our result in Section 2 looks rather classical, it does not seem to have appeared in the literature including the well known works of Walk (1977), Jakubowski (1980); see also the references in Merlevède (2003).

The main purpose of the paper, which will be presented in Section 4, is to consider some goodness of fit tests for diffusion processes. Despite the fact that in the last thirty years diffusion process models have been proved to be immensely useful, the problem of goodness of fit tests for the processes has still been a new issue in recent years. Kutoyants (2004) discussed some possibilities of the construction of such tests in his Section 5.4, where he considered the Kolmogorov-Smirnov statistics based on the continuous observation of a diffusion process. However, his test is not asymptotically distribution free. Negri and Nishiyama (2009) proposed some asymptotically distribution free tests but their results are based on *continuous time observation* of the diffusion processes. One of the interesting points of this paper is that the proposed tests are based on *discrete time observation*, which is more realistic in applications.

Now let us describe the problems treated in Section 4. Consider a one-dimensional stochastic differential equation (SDE)

$$X_t = X_0 + \int_0^t S(X_s)ds + \int_0^t \sigma(X_s)dW_s,$$

where the initial value X_0 is finite almost surely, S and σ are functions which satisfy some properties, and $t \rightsquigarrow W_t$ is a standard Wiener process defined on a stochastic basis $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,\infty)}, P)$. We consider a case where a unique strong solution X to this SDE exists, and we shall assume that X is stationary and ergodic. We are interested in two kinds of goodness of fit test problems:

Problem 1: $H_0 : S = S_0$ versus $H_1 : S \neq S_0$ for a given S_0, with σ being a known function;

Problem 2: $H_0 : \sigma = \sigma_0$ versus $H_1 : \sigma \neq \sigma_0$ for a given σ_0, with S being a known function.

The meaning of the alternatives "$S \neq S_0$" and "$\sigma \neq \sigma_0$" will be precisely stated in the main theorems.

We suppose that the process X is observed at time points $0 = t_0^n < t_1^n < \cdots < t_n^n$ such that

$$t_n^n \to \infty, \quad n\Delta_n^2 \to 0, \quad \text{where } \Delta_n = \max_{1\leq i\leq n} |t_i^n - t_{i-1}^n|,$$

as $n \to \infty$. We will propose some asymptotically distribution free tests based on this sampling scheme, namely, *high frequency data*. We should mention that there is a huge literature on discrete time approximations of statistical estimators for diffusion processes; see e.g. the Introduction of Gobet *et al.* (2004) for a review including not only high frequency cases but also low frequency cases. However, it seems difficult to obtain asymptotically distribution free results based on low frequency data. Our result for the problem 1 improves the preceding work of Masuda *et al.* (2008) who considered some Kolmogorov-Smirnov type tests. The problem 2 is newly considered in this paper. As we will announce in Section 3, the ideal assertion for the Kolmogorov-Smirnov type tests is still an *open* problem because it needs a weak convergence theorem in $\ell^\infty(\mathbb{R})$ space.

参考文献

Dedecker, J. and Merlevède, F. (2003). The conditional central limit theorem in Hilbert spaces. *Stochastic Process. Appl.* **108**, 229-262.

Gobet, E., Hoffmann, M. and Reiss, M. (2004). Nonparametric estimation of scalar diffusions based on low frequency data. *Ann. Statist.* **32**, 2223-2253.

Hoffman-Jorgensen, J. and Pisier, G. (1976). The law of large numbers and the central limit theorem in Banach spaces. *Ann. Probab.* **4**, 587-599.

Jakubowski, A. (1980). On limit theorems for sums of dependent Hilbert space valued random variables. In *Lecture Notes in Statistics* **2**, 178-187. Springer-Verlag, New York.

Kutoyants, Yu.A. (2004). *Statistical Inference for Ergodic Diffusion Processes.* Springer, London.

Masuda, H., Negri, I. and Nishiyama, Y. (2008). Goodness of fit test for ergodic diffusions by discrete time observations: an innovation martingale approach. *Research Memorandum* **1069**, Inst. Statist. Math., Tokyo.

Merlevède, F. (2003). On the central limit theorem and its weak invariance principle for strongly mixing sequences with values in a Hilbert space via martingale approximation. *J. Theoret. Probab.* **16** 625-653.

Negri, I. and Nishiyama, Y. (2009). Goodness of fit test for ergodic diffusion processes. *Ann. Inst. Statist. Math.* **61**, 919-928.

van der Vaart, A.W. and Wellner, J.A. (1996). *Weak Convergence and Empirical Processes: With Applications to Statistics.* Springer-Verlag, New York.

Walk, H. (1977). An invariance principle for the Robbins-Monro process in a Hilbert space. *Z. Wahrsch. Verw. Gebiete* **39**, 135-150.