システム制御理論の研究 ～ 統計科学と制御科学の接点

宮里　義彦　運営企画本部　特任教授

【Model-Based ControlからModel-Free Controlへ】

- Non-Model-Based Control Scheme
  - ⋆モデルに基づく精緻な解析を含まないアプローチ（理論以前）
- Model-Based Control Scheme
  - ⋆モデルに基づく精緻な解析を含むアプローチ，モデル依存度大
- Model-Free Control Scheme
  - ⋆モデルに基づく精緻な解析を含むアプローチ，モデル依存度小
    モデル次数に依存しない，モデル相対次数に依存／依存度小

【適応型サーボ制御問題／問題設定／制御目的】

- 内部モデル原理に基づくサーボ制御（追値制御と外乱抑制）

制御対象
$\frac{d}{dt}x(t) = Ax(t) + bu(t) + gd_1(t)$ $y(t) = c^T x(t) + d_2(t)$ $e(t) = y_M(t) - y(t)$
$y \in \mathbf{R}$ (出力), $u \in \mathbf{R}$ (制御入力), $x \in \mathbf{R}^n$ (状態変数) $y_M \in \mathbf{R}$ (目標信号), $e \in \mathbf{R}$ (制御誤差), $d_1, d_2 \in \mathbf{R}$ (外乱)

- 適応型サーボ制御の問題設定と制御目的

問題設定（仮定）
入力$u(t)$, 出力$y(t)$, 目標信号$y_M(t)$は既知
状態$x(t)$，システムA, b, c, g, 外乱$d_1(t)$, $d_2(t)$, 次数nは未知
$A_d(s)d_i(t) = 0$, $A_d(s)y_M(t) = 0$ (線形自由系による生成)
多項式$A_d(s)$は未知，$A_d(s)$の次数の上界 (n_d) は既知
相対次数n^*は既知 **(制御系 I)** ($n^* \equiv \min_{1 \le i \le n}(i : c^T A^{i-1} b \neq 0)$)
相対次数n^*はr, $r+1$, $r+2$のいずれかでrは既知 **(制御系 II)**
制御目的
$e(t) \to 0$ (目標信号へのトラッキングと外乱の影響の抑制)

【適応型サーボ制御系I】(適応的な内部モデル原理)

表 1. 次数に依存しない適応サーボ制御器（既知の相対次数）

$$
\begin{aligned}
&z_1(t) \equiv e(t) = y(t) - y_M(t)\\
&z_i(t) \equiv u_{fn^*-i+1}(t) - \alpha_{i-1}(t) \qquad (i = 2, \cdots, n^*)
\end{aligned}
$$

$$
\begin{aligned}
\alpha_1(t) &= -\hat{p}(t)\hat{\phi}(t) - \hat{k}_1(t)z_1(t),\\
\alpha_2(t) &= \lambda u_{f2}(t) + \beta_1(t) + \gamma_1(t)\hat{\Theta}(t)^T\omega_1(t)\\
&\quad +\hat{b}_0(t)\{\gamma_1(t)u_{fn^*-1}(t) - z_1(t)\}\\
&\quad -k_{21}z_2(t) - k_{22}\gamma_1(t)^2 z_2(t) + \gamma_{\theta 1}(t)\tau_{\theta 2}(t)\\
\alpha_i(t) &= \lambda u_{fn^*-i+1}(t) + \beta_{i-1}(t) + \gamma_{i-1}(t)\hat{\Theta}(t)^T\omega_1(t)\\
&\quad +\hat{b}_0(t)\gamma_1(t)u_{fn^*-1}(t) - z_{i-1}(t)\\
&\quad -k_{i1}z_i(t) - k_{i2}\gamma_{i-1}(t)^2 z_i(t) + \gamma_{\theta i-1}(t)\tau_{\theta i}(t)\\
&\quad +\gamma_{bi-1}(t)\tau_{bi}(t) + \tilde{\alpha}_{i-1}(t) \quad (i = 3, \cdots, n^*)
\end{aligned}
$$

$$
\begin{aligned}
\beta_{i-1}(t) &= \frac{\partial\alpha_{i-1}}{\partial\hat{K}_1}\dot{\hat{K}}_1(t) + \frac{\partial\alpha_{i-1}}{\partial\omega_{i-1}}\dot{\omega}_{i-1}(t) - \frac{\partial\alpha_{i-1}}{\partial z_1}\lambda z_1(t)\\
&\quad + \textstyle\sum_{j=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial z_j}\{-\lambda u_{fn^*-j+1}(t) + u_{fn^*-j}(t) - \beta_{j-1}(t)\}\\
\tilde{\alpha}_i(t) &= -\textstyle\sum_{k=2}^{i-1}\gamma_{\theta k-1}(t)G_{13}\gamma_{i-1}(t)\omega_1(t)z_k(t)\\
&\quad -\textstyle\sum_{k=3}^{i-1}\gamma_{bk-1}(t)g_2\gamma_{i-1}(t)u_{fn^*-1}(t)z_k(t)\\
\omega_{i-1}(t) &= [u_{fn^*-i+2}(t),\ \omega_{i-2}(t)^T]^T\\
\gamma_{i-1}(t) &= \frac{\partial\alpha_{i-1}}{\partial z_1} - \textstyle\sum_{j=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial z_j}\gamma_{j-1}(t)\\
\gamma_{\theta i-1}(t) &= \frac{\partial\alpha_{i-1}}{\partial\hat{\Theta}} - \textstyle\sum_{j=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial z_j}\gamma_{\theta j-1}(t)\\
\gamma_{b2}(t) &= \frac{\partial\alpha_2}{\partial\hat{b}_0}\\
\gamma_{bi-1}(t) &= \frac{\partial\alpha_{i-1}}{\partial\hat{b}_0} - \textstyle\sum_{j=3}^{i-1}\frac{\partial\alpha_{i-1}}{\partial z_j}\gamma_{bj-1}(t)
\end{aligned}
$$

$$
\begin{aligned}
\tau_{\theta 1}(t) &\equiv G_{13}\omega_1(t)z_1(t)\\
\tau_{\theta i}(t) &= \tau_{\theta i-1}(t) - G_{13}\gamma_{i-1}(t)\omega(t)_1 z_i(t)\\
\tau_{b2}(t) &= g_2\{z_1(t) - \gamma_1(t)u_{fn^*-1}(t)\}z_2(t)\\
\tau_{bi}(t) &= \tau_{bi-1}(t) - g_2\gamma_{i-1}(t)u_{fn^*-1}z_i(t) \qquad (i = 3, \cdots, n^*)
\end{aligned}
$$

Control Law and Adaptive Law :

$$
\begin{aligned}
&u(t) = \alpha_{n^*}(t)\\
&\dot{\hat{k}}_1(t) = g_{11}z_1(t)^2, \quad \dot{\hat{p}}(t) = g_{12}\hat{\phi}(t)z_1(t)\\
&\dot{\hat{\Theta}}(t) = \tau_{\theta n^*}(t), \quad \dot{\hat{b}}(t) = \tau_{bn^*}(t)
\end{aligned}
$$

【適応型サーボ制御系II】(適応的な内部モデル原理)

- 強正実有理関数の相対次数の自由度 (-1, 0, +1) の利用

表 2. 適応則 ⇔ 誤差方程式 (相対次数 (-1, 0, +1)) ⇔ 正定関数

$$
\begin{aligned}
&\dot{\hat{\theta}}(t) = \omega(t)e(t)\\
&\Leftrightarrow \begin{cases}
\left(\frac{d}{dt} + \lambda\right)e(t) = \{\theta - \hat{\theta}(t)\}\omega(t)\\
e(t) = \{\theta - \hat{\theta}(t)\}\omega(t)\\
e(t) = \left(\frac{d}{dt} + \lambda\right)\{\theta - \hat{\theta}(t)\}\omega(t)
\end{cases}\\
&\Leftrightarrow \begin{cases}
V(t) = e(t)^2 + \{\theta - \hat{\theta}(t)\}^2\\
V(t) = \{\theta - \hat{\theta}(t)\}^2\\
V(t) = e_f(t)^2 + \{\theta - \hat{\theta}(t)\}^2 \quad (\dot{e}_f + \lambda e_f = e)
\end{cases}\\
&\Leftrightarrow \begin{cases}
\frac{d}{dt}V(t) = -2\lambda e(t)^2 \le 0\\
\frac{d}{dt}V(t) = -2e(t)^2 \le 0\\
\frac{d}{dt}V(t) = -2\lambda e_f(t)^2 \le 0
\end{cases}
\end{aligned}
$$

表 3. 次数に依存しない適応サーボ制御器（不確定な相対次数）

$$
\begin{aligned}
&z_1(t) \equiv e(t) = y(t) - y_M(t)\\
&z_i(t) \equiv u_{fr-i+2}(t) - \alpha_{i-1}(t) \quad (i = 2, \cdots, r+1)
\end{aligned}
$$

$$
\begin{aligned}
\alpha_1(t) &= -\hat{k}_1(t)z_1(t) - \hat{\Phi}_1(t)^T\omega_1(t)\\
\alpha_2(t) &= \hat{k}_{20}(t)\{\lambda u_{fr}(t) + \beta_1(t)\} - \hat{k}_{21}(t)z_2(t)\\
&\quad -k_{22}\gamma_1(t)^2 z_2(t) - \hat{\Phi}_0(t)^T\omega_0(t)\\
\alpha_i(t) &= \lambda u_{fr-i+2}(t) + \beta_{i-1}(t) - z_{i-1}(t)\\
&\quad +\gamma_{i-1}(t)\hat{B}_0(t)^T\tilde{\omega}_0(t) - k_{i1}z_i(t) - k_{i2}\gamma_{i-1}(t)^2 z_i(t)\\
&\quad +\gamma_{Bi-1}(t)\tau_{Bi}(t) + \tilde{\alpha}_i(t) \quad (i = 3, \cdots, r+1)
\end{aligned}
$$

$$
\begin{aligned}
\beta_1(t) &= \frac{\partial\alpha_1}{\partial\omega_1}\dot{\omega}_1(t) - \frac{\partial\alpha_1}{\partial z_1}\lambda z_1(t)\\
\beta_{i-1}(t) &= \frac{\partial\alpha_{i-1}}{\partial\hat{K}_2}\dot{\hat{K}}_2(t) + \frac{\partial\alpha_{i-1}}{\partial u_f^{(r-i+3)}}\dot{u}_f^{(r-i+3)}(t)\\
&\qquad -\frac{\partial\alpha_{i-1}}{\partial z_1}\lambda z_1(t)\\
&\qquad + \textstyle\sum_{j=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial z_j}\{-\lambda u_{fr-j+2}(t)\\
&\qquad +u_{fr-j+1}(t) - \beta_{j-1}(t)\} \quad (i = 3, \cdots, r+1)\\
\tilde{\alpha}_i(t) &= -\textstyle\sum_{k=4}^{i-1}\gamma_{Bk-1}(t)G_3\gamma_{i-1}(t)\tilde{\omega}_0(t)z_k(t)\\
\tilde{\omega}_0(t) &= [u_{fr-1}(t),\ \omega_0(t)^T]^T\\
&= [u_{fr-1}(t),\ u_{fr}(t),\ \cdots,\ u_{fr+n_d-1}(t)]^T\\
u_f^{(r-i+3)}(t) &= [u_{fr-i+3}(t),\ u_{fr-i+4}(t),\ \cdots,\ u_{fr+n_d+1}(t)]^T\\
\gamma_{i-1}(t) &= \frac{\partial\alpha_{i-1}}{\partial\hat{K}_1}G_1 v_1(t) + \frac{\partial\alpha_{i-1}}{\partial z_1} - \textstyle\sum_{j=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial z_j}\gamma_{j-1}(t)\\
\gamma_{B2}(t) &= 0, \quad \gamma_{Bi-1}(t) = \frac{\partial\alpha_{i-1}}{\partial\hat{B}_0} - \textstyle\sum_{j=4}^{i-1}\frac{\partial\alpha_{i-1}}{\partial z_j}\gamma_{Bj-1}(t)\\
&\qquad (i = 4, \cdots, r+1)
\end{aligned}
$$

$$
\begin{aligned}
\tau_{B3}(t) &= -G_3\gamma_2(t)\tilde{\omega}_0(t)z_3(t)\\
\tau_{Bi}(t) &= \tau_{Bi-1}(t) - G_3\gamma_{i-1}(t)\tilde{\omega}_0(t)z_i(t) \quad (i = 4, \cdots, r+1)
\end{aligned}
$$

Control Law :

$$u(t) = \alpha_{r+1}(t)$$

Adaptive Law :

$$
\begin{aligned}
&\hat{k}_1(t) = \hat{k}_{11}(t) + \hat{k}_{12}(t)\\
&\quad \hat{k}_{11}(t) = \tfrac{1}{2}g_{11}z_1(t)^2, \quad \dot{\hat{k}}_{12}(t) = \lambda g_{11}z_1(t)^2\\
&\quad \Leftrightarrow\ \dot{\hat{k}}_1(t) = g_{11}z_1(t)Z_1(t)\\
&\hat{\Phi}_1(t) = \hat{\Phi}_{11}(t) + \hat{\Phi}_{12}(t)\\
&\quad \hat{\Phi}_{11}(t) = G_{12}\omega_1(t)z_1(t)\\
&\quad \dot{\hat{\Phi}}_{12}(t) = G_{12}\{\lambda\omega_1(t) - \dot{\omega}_1(t)\}z_1(t)\\
&\quad \Leftrightarrow\ \dot{\hat{\Phi}}_1(t) = G_{12}\omega_1(t)Z_1(t)\\
&\dot{\hat{k}}_{20}(t) = -g_{20}\{\lambda u_{fr}(t) + \beta_1(t)\}z_2(t)\\
&\dot{\hat{k}}_{21}(t) = g_{21}z_2(t)^2\\
&\dot{\hat{\Phi}}_0(t) = G_{22}\omega_0(t)z_2(t)\\
&\dot{\hat{B}}_0(t) = \tau_{Br+1}(t)
\end{aligned}
$$