

Extensions of the ETAS model

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Space time ETAS model

The Epidemic-Type Aftershock Sequence (ETAS) model has been widely and successfully used to quantify the clustering patterns of seismicity. Its early version only focused on earthquake occurrence time (Ogata 1988, 1989, 1992) and was generalized by Ogata (1998) to the space–time ETAS model by incorporating both the earthquake locations and occurrence times. This version and its variations became the standard model in data. This poster summarizes the extensions of the ETAS models.

Conditional intensity of the model

$$\lambda(t, x, y) = \mu(x, y) + \sum_{i: t_i < t} \kappa(m_i) g(t - t_i) f(x - x_i, y - y_i; m_i)$$

Productivity, average # of triggered events (Yamanaka and Shimazaki, 1990)

$$\kappa(m) = A \exp[\alpha(m - m_0)]$$

Time p.d.f. of triggered events, Omori-Utsu formula (Omori, 1898; Utsu, 1957)

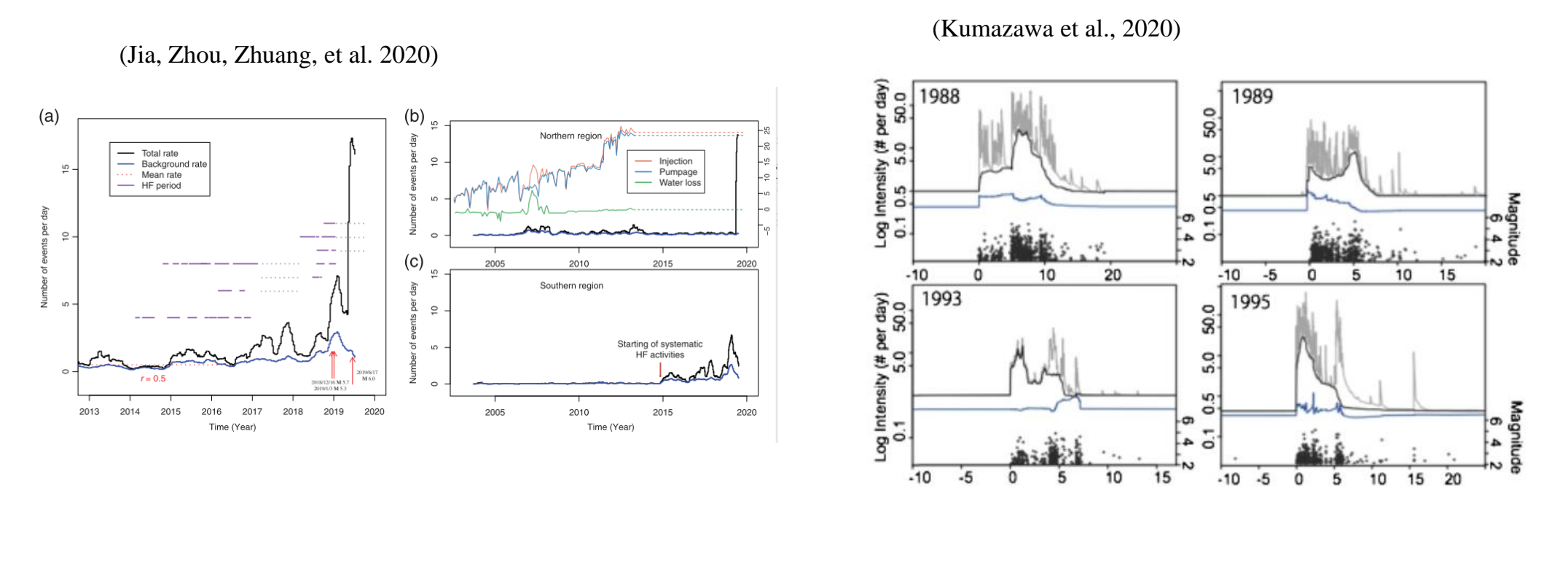
$$g(t) = \frac{p-1}{c} (1+t)^{-p}; \quad t > 0$$

P.d.f. of spatial location from an ancestor of m (Zhuang et al 2005, Ogata & Zhuang 2006)

$$f(x, y; m) = \frac{q-1}{\pi D e^{\gamma m}} \left(1 + \frac{x^2 + y^2}{D e^{\gamma m}} \right)^{-q}$$

ETAS model with non-stationary background

1. Early work (Tsukakoshi & Shimazaki, 2006) (Lombardi & Marzocchi, 2007)
- Stochastic reconstruction based histogram: testing background stationarity
2. Kernel estimates (Jia, Zhou, Zhuang, et al. 2020)
- $$\hat{\mu}(t, x, y) = \frac{1}{T} \sum \varphi_i Z_1(t - t_i; h_i^{(t)}) Z_2(x - x_i, y - y_i; h_i^{(s)}),$$
- Stochastic reconstruction based histogram:
- Application in induced seismicity
3. Linear interpolation with Bayesian smoothness prior (Kumazawa et al., 2020)
- $$\Phi_{\mu} = \sum_{i=0}^N \left(\frac{q_{\mu,i+1} - q_{\mu,i}}{t_{i+1} - t_i} \right)^2 (t_{i+1} - t_i)$$
- Application volcanic seismicity

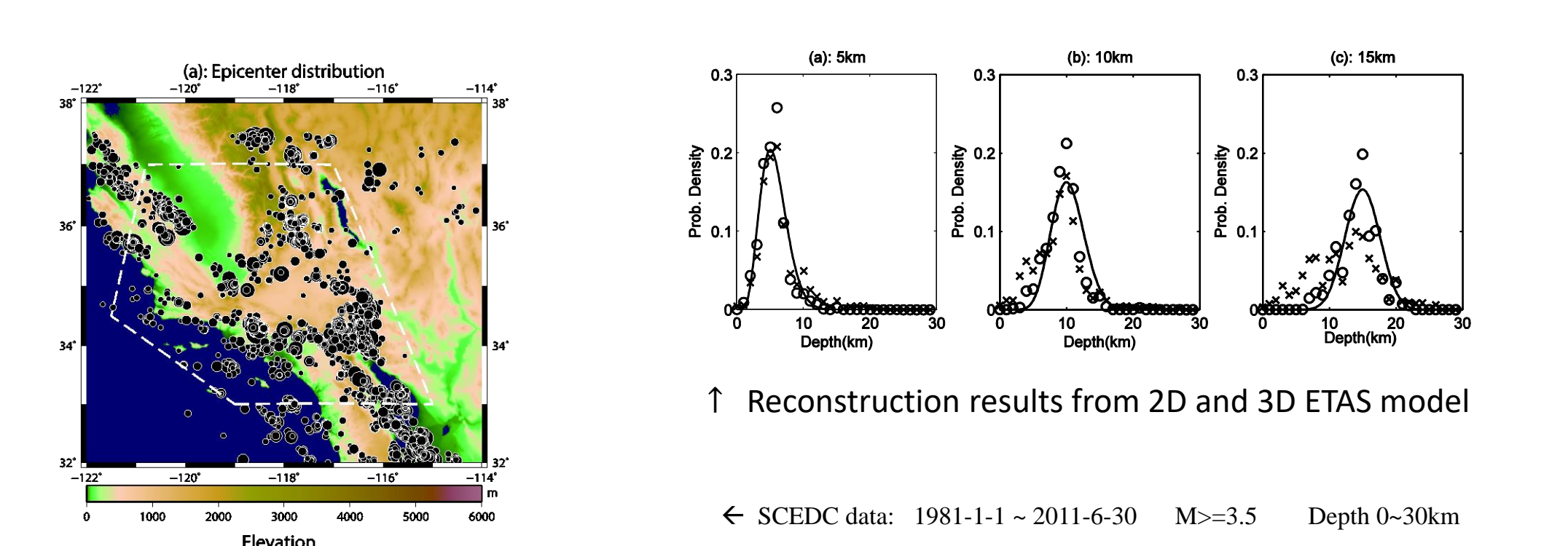


Hypocentral 3D ETAS model (Guo, Zhuang & Zhou, 2015, GJI)

$$\lambda(t, x, y, z) = \mu(x, y, z) + \sum_{i: t_i < t} \kappa(m_i) g(t - t_i) f(x - x_i, y - y_i; m_i) h(z, z_i)$$

$$h(z; z_i) \propto \left(\frac{z}{Z}\right)^{\eta \frac{z_i}{Z}} \left(1 - \frac{z}{Z}\right)^{\eta \left(1 - \frac{z_i}{Z}\right)}, \quad Z: \text{maximum depth}$$

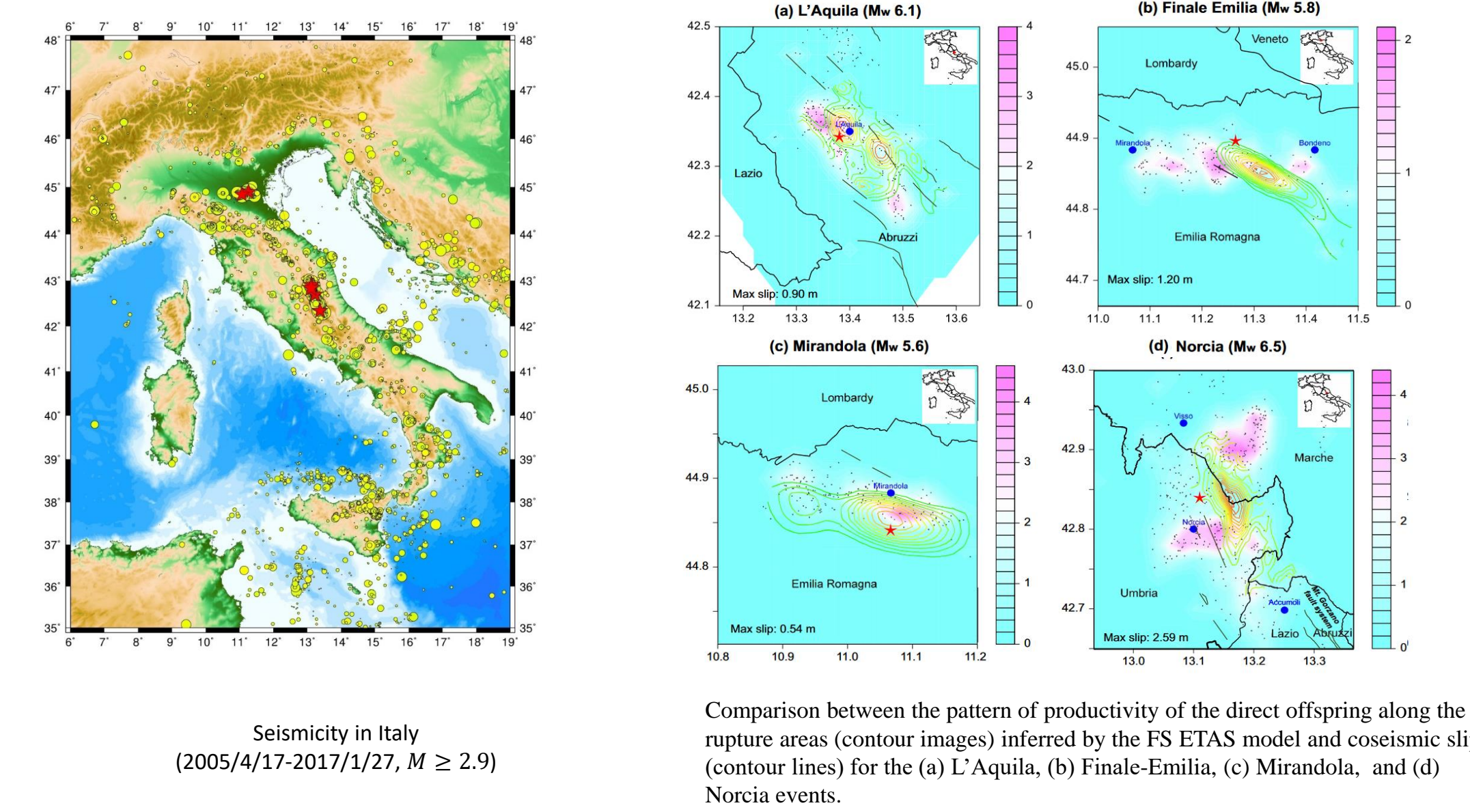
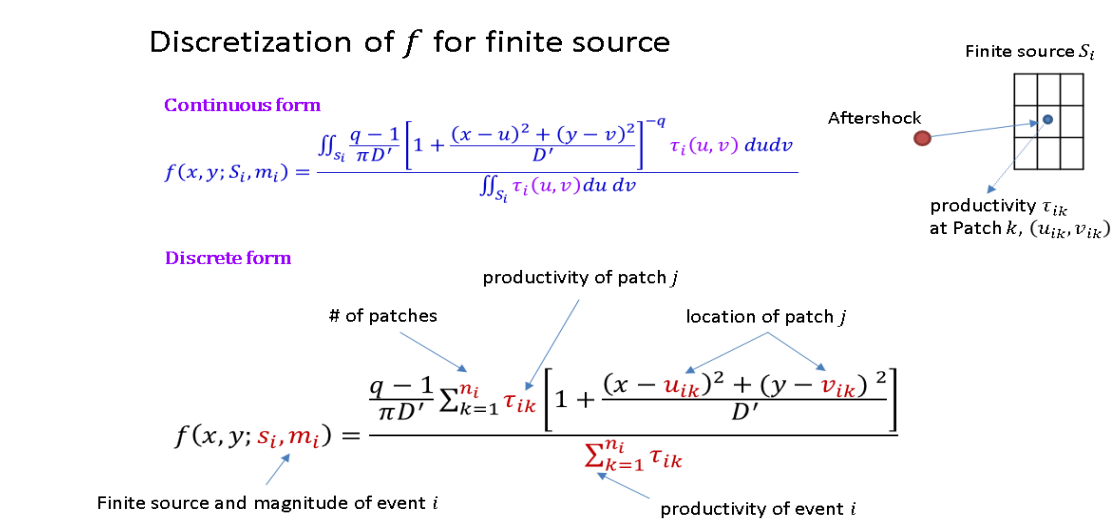
Models	logL
Epicentral ETAS model + uniform depth distribution	-4761.4
Epicentral ETAS model + empirical depth distribution	-3454.3
Hypocentral (3D) ETAS model	-2904.6



2D finite source ETAS model (Guo, Zhuang & Zhou, 2015, JGR)

$$\lambda(t, x, y) = \mu(x, y) + \sum_{i: t_i < t} \kappa(m_i) g(t - t_i) f(x - x_i, y - y_i; S_i, m_i)$$

$$f(x, y; S_i, m_i) = \begin{cases} \frac{q-1}{\pi D e^{\gamma(m_i-m_0)}} \left(1 + \frac{x^2 + y^2}{D e^{\gamma(m_i-m_0)}} \right)^{-q}, & \text{if } S_i \text{ is point source} \\ \frac{\iint_{S_i} \frac{q-1}{\pi D'^2} \left[1 + \frac{(x-u)^2 + (y-v)^2}{D'^2} \right]^{-q} \tau_i(u, v) du dv}{\iint_{S_i} \tau_i(u, v) du dv}, & \text{if } S_i \text{ is finite source} \end{cases}$$

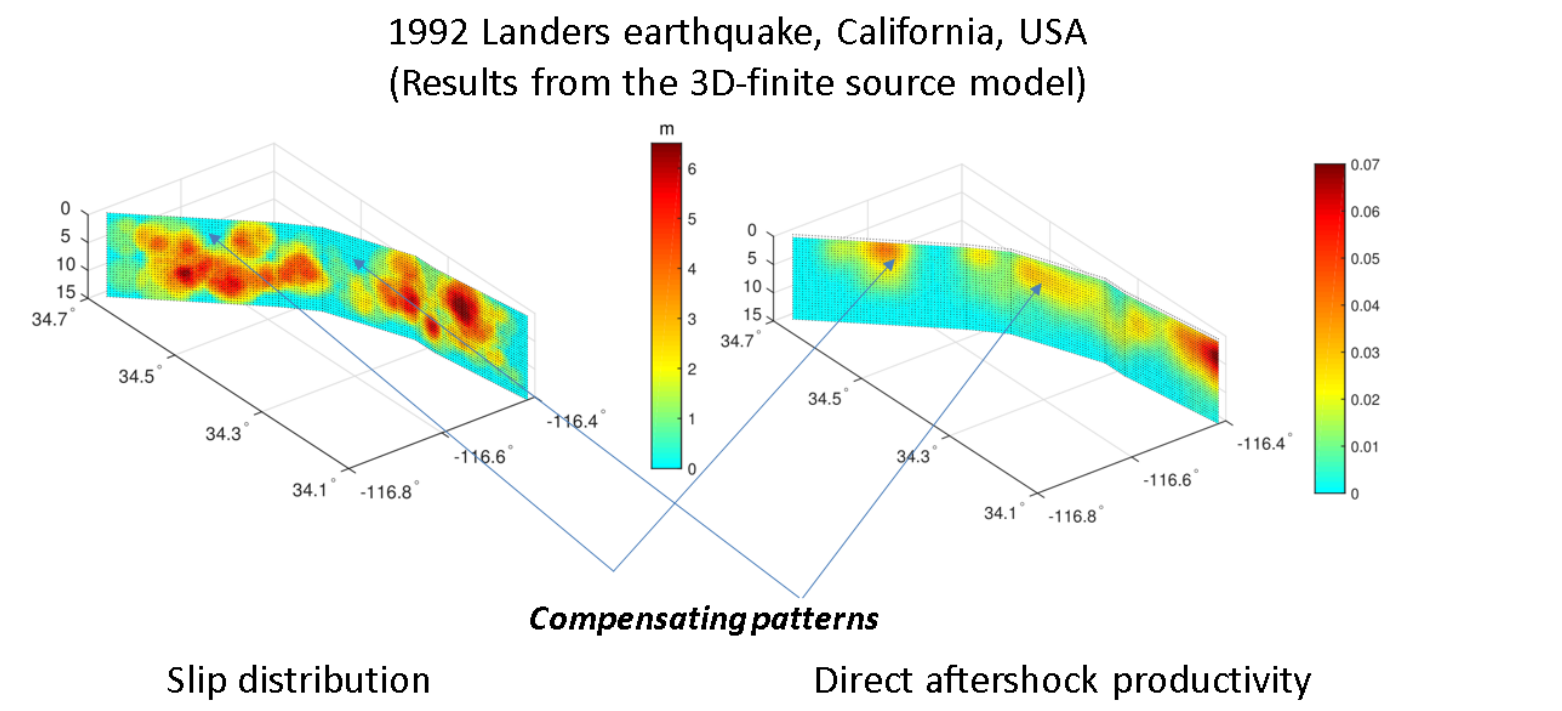


3D finite source ETAS model (Guo, Zhuang & Zhang, 2021, JGR)

$$\lambda(t, x, y, z) = \mu(x, y, z) + \sum_{i: t_i < t} \kappa(m_i) g(t - t_i) f(x, y, z; S_i, m_i)$$

$$f(x, y, z; S_i, m_i) = \begin{cases} \frac{h(z; z_i)}{\pi D e^{\gamma(m_i-m_0)}} \left(1 + \frac{(x-x_i)^2 + (y-y_i)^2}{D e^{\gamma(m_i-m_0)}} \right)^{-q}, & \text{if } S_i \text{ is point source} \\ \frac{q-1}{\pi D'^2} \iiint_{S_i} \left[1 + \frac{(x-w)^2 + (y-v)^2}{D'^2} \right]^{-q} h(z; w) \tau_i(u, v, w) dudvdw}{\iiint_{S_i} \tau_i(u, v, w) dudvdw}, & \text{if } S_i \text{ is finite source} \end{cases}$$

$$h(z; z_i) \propto \left(\frac{z}{Z}\right)^{\eta \frac{z_i}{Z}} \left(1 - \frac{z}{Z}\right)^{\eta \left(1 - \frac{z_i}{Z}\right)} \quad \frac{z}{Z} \mid z_i \sim \text{Beta} \left(\eta \frac{z_i}{Z} + 1, \eta \left(1 - \frac{z_i}{Z}\right) + 1 \right) \quad Z: \text{maximum depth}$$



ETAS model + focal Mechanisms

$$\lambda(t, x, y, \phi) = \mu(x, y) \zeta(\Delta(\phi, \phi_0(x, y))) + \sum_{i: t_i < t} \kappa(m_i) g(t - t_i) f(x - x_i, y - y_i; m_i) \xi(\Delta(\phi, \phi_i))$$

Rotation poles are regarded as uniformly distributed.

