経験的確率モデルによる台風の気候的特性の解析

中野 慎也　モデリング研究系 准教授

Introduction

- It is important to evaluate the statistical (climatological) properties of typhoon in order to assess the risks of typhoon hazards.
- In particular, climatic variations of typhoons is an important issue.
- Since the number of typhoons in each year is small, it is difficult to distinguish between climatic variations and random properties of typhoons.
- In order to enable us to evaluate climatic variations of typhoons, we are developing an experimental framework based on a data-driven stochastic simulator, which is obtained by statistical analysis of the data of past typhoon trajectories for more than sixty years.
- In this study, we examine the effects of inter-annual variations of typhoon genesis and translational motion on the frequency of approaching the main islands of Japan.

Data

We use the best track data provided by Japan Meteorological Agency (JMA), which are available since 1951. This data set contains the records about the position, pressure at the centre, wind speed, and so on. In this study, we use the data of the position and the pressure at the centre of the typhoons.

Typhoon's translation velocity (per hour) is obtained by the difference in position between two sequential time steps:

$$\boldsymbol{v}(z_{n,i}) = \left(\boldsymbol{x}(z_{n,i+1}) - \boldsymbol{x}(z_{n,i})\right)/6$$

(n: identifier for a typhoon; i: time step).

Gaussian process regression

We assume that typhoon translation velocity $\boldsymbol{v}$ is represented as a function of latitude λ, longitude ϕ, day of year d, and Julian day Y. We hereinafter denote the collection of these four variable as z as follows:

$$\boldsymbol{z} = (\phi \;\; \lambda \;\; d \;\; Y)^T.$$

Thus, the observed translation velocity of a typhoon n at a time step i can be written as $\boldsymbol{v}^o_{n,i} = \boldsymbol{v}(\boldsymbol{z}^o_{n,i}) + \varepsilon_{n,i}$.

The relationship of the translation velocity $\boldsymbol{v}$ with the four input variables is modeled using the Gaussian process regression (e.g., Rasmussen and Williams, 2006). Here, the following Gaussian process is considered as a prior:

$$\boldsymbol{v} \sim \mathcal{GP}\left(0; \xi^2 k(\boldsymbol{z}, \boldsymbol{z}')\right).$$

The mean and variance of the posterior distribution for each variable given all the records $\mathsf{V}^o = \left(\boldsymbol{v}^o_{1,1} \cdots \boldsymbol{v}^o_{N,T_N}\right)$ are then obtained in the following form:

$$E[v(\boldsymbol{z}^*_i) \,|\, \boldsymbol{V}^o] = \boldsymbol{V}^{oT}(\mathsf{K} + \lambda^2 \mathsf{I})^{-1} \boldsymbol{k}(\boldsymbol{z}^*_i),$$
$$V[v(\boldsymbol{z}^*_i) \,|\, \boldsymbol{V}^o] = \sigma_V^2 \left[k(\boldsymbol{z}^*_i, \boldsymbol{z}^*_i) - \boldsymbol{k}^T(\boldsymbol{z}^*_i)(\mathsf{K} + \lambda^2 \mathsf{I})^{-1} \boldsymbol{k}(\boldsymbol{z}^*_i)\right],$$

where $\boldsymbol{k}$ and K are defined as follows:

$$\boldsymbol{k}(\boldsymbol{z}) = \left(k(\boldsymbol{z}, \boldsymbol{z}_{1,1}) \;\cdots\; k(\boldsymbol{z}, \boldsymbol{z}_{N,T_N})\right)^T, \quad \mathsf{K} = \begin{bmatrix} k(\boldsymbol{z}_{1,1}, \boldsymbol{z}_{1,1}) & \cdots & k(\boldsymbol{z}_{1,1}, \boldsymbol{z}_{N,T_N}) \\ \vdots & \ddots & \vdots \\ k(\boldsymbol{z}_{N,T_N}, \boldsymbol{z}_{1,1}) & \cdots & k(\boldsymbol{z}_{N,T_N}, \boldsymbol{z}_{N,T_N}) \end{bmatrix}.$$

We use this posterior mean as a model of the typhoon translation velocity.

Parameter estimation

The covariance function $\xi^2 k(\boldsymbol{z}, \boldsymbol{z}')$ is given so as to satisfy:

$$k(\boldsymbol{z}, \boldsymbol{z}') = \exp\left[-\frac{(\phi - \phi')^2}{\sigma_\phi^2} - \frac{(\lambda - \lambda')^2}{\sigma_\lambda^2} - \frac{(d - d')^2}{\sigma_d^2} - \frac{(Y - Y')^2}{\sigma_Y^2}\right].$$

We assume $\sigma_Y = 4$. The parameters $\sigma_\phi, \sigma_\lambda, \sigma_d$ and ξ were determined using the 5-fold cross validation where the data set was divided according to the year of occurrence.

(The leave-one-out cross validation would remarkably underestimate the values of σ_d, σ_Y, because the velocity at a certain time usually takes a very similar value to the mean of the velocity between the previous and subsequent times of the same typhoon.)

References

C. E. Rasmussen and C. K. I. Williams: "Gaussian processes for machine learning", the MIT press, 2006.

S. Nakano, K. Ito, K. Suzuki, and G. Ueno: "Decadal-scale meridional shift of the typhoon recurvature latitude over five decades", *Int. J. Climatol.*, v. 36, pp. 3819-3827, doi:10.1002/joc.4595, 2016.

Sampling from the empirical simulator

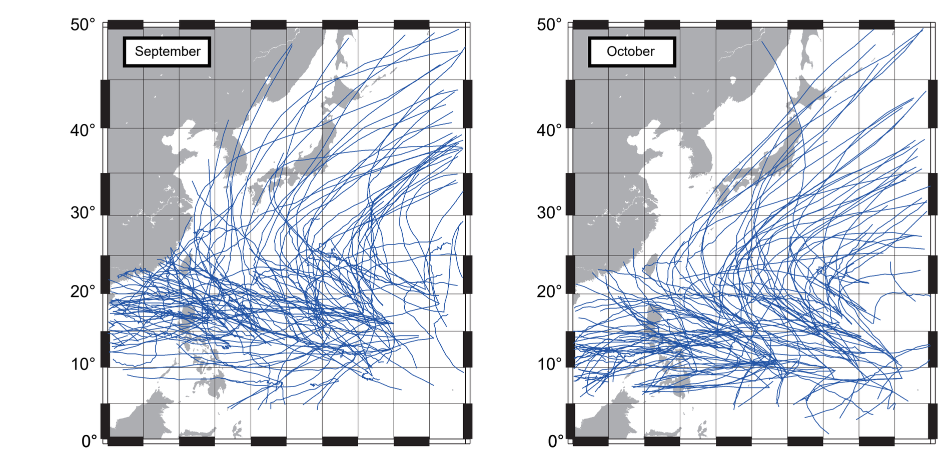
If we already obtained the translation velocities at the points $\{\boldsymbol{x}^*_0, \boldsymbol{x}^*_1, \ldots, \boldsymbol{x}^*_k\}$. and if we define a vector $\boldsymbol{V}_k$ and a matrix K_k as follows:

$$\boldsymbol{V}_k = \begin{pmatrix} v(\boldsymbol{z}^O_1) \\ \vdots \\ v(\boldsymbol{z}^O_N) \\ v(\boldsymbol{z}^*_0) \\ \vdots \\ v(\boldsymbol{z}^*_k) \end{pmatrix}, \quad \mathsf{K}_k = \begin{bmatrix} k(\boldsymbol{z}^O_1, \boldsymbol{z}^O_1) & \cdots & k(\boldsymbol{z}^O_1, \boldsymbol{z}^O_N) & k(\boldsymbol{z}^O_1, \boldsymbol{z}^*_0) & \cdots & k(\boldsymbol{z}^O_1, \boldsymbol{z}^*_k) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ k(\boldsymbol{z}^O_N, \boldsymbol{z}^O_1) & \cdots & k(\boldsymbol{z}^O_N, \boldsymbol{z}^O_N) & k(\boldsymbol{z}^O_N, \boldsymbol{z}^*_0) & \cdots & k(\boldsymbol{z}^O_N, \boldsymbol{z}^*_k) \\ k(\boldsymbol{z}^*_0, \boldsymbol{z}^O_1) & \cdots & k(\boldsymbol{z}^*_0, \boldsymbol{z}^O_N) & k(\boldsymbol{z}^*_0, \boldsymbol{z}^*_0) & \cdots & k(\boldsymbol{z}^*_0, \boldsymbol{z}^*_k) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ k(\boldsymbol{z}^*_k, \boldsymbol{z}^O_1) & \cdots & k(\boldsymbol{z}^*_k, \boldsymbol{z}^O_N) & k(\boldsymbol{z}^*_k, \boldsymbol{z}^*_0) & \cdots & k(\boldsymbol{z}^*_k, \boldsymbol{z}^*_k) \end{bmatrix},$$

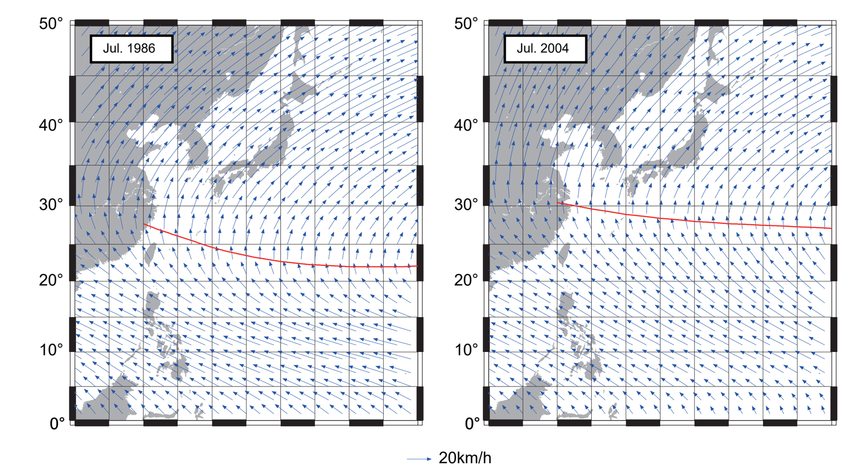
the translation velocity for the next point $\boldsymbol{z}^*_{k+1}$ becomes a Gaussian distribution with the following mean and variance:

$$E[v(\boldsymbol{z}^*_{k+1}) \,|\, \boldsymbol{V}_k] = \boldsymbol{V}_k^T(\mathsf{K}_k + \lambda^2 \mathsf{I})^{-1} \boldsymbol{k}(\boldsymbol{z}^*_{k+1}),$$
$$V[v(\boldsymbol{z}^*_{k+1}) \,|\, \boldsymbol{V}_k] = \sigma_V^2 \left[k(\boldsymbol{z}^*_{k+1}, \boldsymbol{z}^*_i) - \boldsymbol{k}^T(\boldsymbol{z}^*_{k+1})(\mathsf{K}_k + \lambda^2 \mathsf{I})^{-1} \boldsymbol{k}(\boldsymbol{z}^*_{k+1})\right].$$

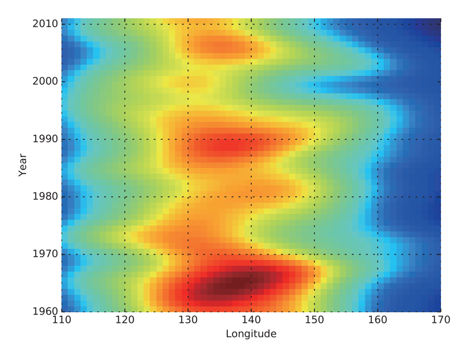
Samples under the condition for 2004 generated from the model



Climatic variations of typhoons



Variations of averaged velocity field of typhoons between July 1986 and July 2004. Red lines indicate the latitudes of the zero east–west velocity (Nakano et al., 2016).



Variations of longitudes of typhoon genesis from 1960 to 2010 which were smoothed using the kernel density estimation technique.