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AR-モデルにおけるロバスト推定のプログラム

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ROBARというALGOLで書かれたプログラムは自己回帰過程のパラメータの最適なロバスト推定量を計算する．未知なパラメータはETA＝データの平均，SGM＝イノベーションの標準偏差TH [1],…, TH [P]＝自己回帰パラメータである．この推定量の定義や効用はKünsch (1983), (1984) にある．

推定量は事前に決めなくてはならない定数 $C1, C2$ に依存する．もし $C1$ が増加すれば，THのロバストネスが減少するけれど，THの推定効率が増加する．$C1$ の選択は，ETAとSGMの推定量の性質に無関係である．同様に，$C2$ はTHの推定量に無関係で，ETAとSGMのロバストネスと効率を調整する．プログラムは，$C1$ をCMINとした場合とCMAXとした場合の2つの場合の計算をする．ここでCMIN（CMAX）は，THの効率が 1.15^{-1}（1.05^{-1}）になる値である．$C2$ も1.0と1.5の2つの場合が求められる．

AR-モデルの次数 P は，1と PMAXの間で変わる．プログラムは，P MAXにどんな値を与えても動く筈であるけれど，P MAX＞10の時の数値安定性やCPU時間について，まだ経験がない．

推定量は，Künsch (1983)の方程式

$$\sum_{i=1}^{n-p} w_{c_1}\{r(x_i-\hat{\xi},\cdots,x_{i+p-1}-\hat{\xi},\hat{\beta})\hat{u}_{i+p}/\hat{\sigma}^2\}(x_{i+h}-\hat{\xi})\hat{u}_{i+p}=0 \quad (h=0,1,\ldots,p-1), \tag{1}$$

$$\sum_{i=1}^{n-p} w_{c_2}(\hat{u}_{i+p}/\hat{\sigma})\hat{u}_{i+p}=0 \tag{2}$$

$$\sum_{i=1}^{n-p} w_{c_2}(\hat{u}_{i+p}/\hat{\sigma})^2\hat{u}_{i+p}^2=(n-p-1)\alpha\hat{\sigma}^2, \tag{3}$$

ここで

$$w_c(x)=\min(1,c/|x|) \qquad (x\in \boldsymbol{R})$$

$$r(z_1,\ldots,z_p,\beta)=\left(\sum_{k=1}^{p}\sum_{j=1}^{p} z_k A_{kj}(\beta) z_j\right)^{1/2}$$

$$\hat{u}_{i+p}=x_{i+p}-\hat{\xi}-\sum_{j=1}^{p}\hat{\beta}_j(x_{i+p-j}-\hat{\xi})$$

$$\alpha=(2\pi)^{-1/2}\int x^2 w_{c_2}(x)^2\exp(-x^2/2)dx$$

の解である．ARPARというPROCEDUREは，ETAとSGMを固定した時の(1)を解く．NUISANCEというPROCEDUREは，THを固定した時の(2)～(3)を解く．収束まで，このふたつのPROCEDUREを繰り返し使っている．初期値は，低い次数の推定値を使ってINITIALというPROCEDUREで計算する．しかし，同じ次数で違う $C1$ か $C2$ の推定値が既に求められている時は，それを使う．

MAXLIKというPROCEDUREは，古典的近似的最尤推定量を計算する．この推定値とロバスト推定値が大きく違っていたら，データの中にモデルに合わない部分がある．例えば，異常

値やTRANSIENTS等である．この部分はCHECKというプログラムで発見できる．

LIN, INCR, MEDIAN, LOCSCL, PARCORは補助のPROCEDUREである．INDATA, OUTCOVとOUTPARは，INPUT/OUTPUT PROCEDUREである．

AR-モデルをあてはめてから，CHECKというプログラムを用いてデータのDIAGNOSEをする．説明のためロバスト推定量を定義する方程式(1)−(3)をみよう．古典的な推定量と違って重み w_{c_1} と w_{c_2} が入る．であるから古典的推定値とロバスト推定値が違うのは，この重みが1より小さくなるようなデータの部分がある場合である．方程式(2)と(3)の重み w_{c_2} は，標準化されたイノベーション $\hat{u}_i/\hat{\sigma}$ にのみ依存する．このイノベーションはINNOVというPROCEDUREが計算する．イノベーションが大きい観測値が含まれていると，ETAとSGMのロバスト推定値と古典的な推定値が違う．しかし，イノベーション $|\hat{u}_i|$ が大きくても必ずしもデータ X_i が異常値であるとはかぎらない．X_i の前のデータ $X[i-1],\ldots,X[i-p]$ の中に異常値がある時も $|\hat{u}_i|$ が大きくなる．だから，CHISQというPROCEDUREは，観測値の部分系列 $X[i-Q],\ldots,X[I-1]$ $(I=Q+1,\ldots,N)$ のカイ二乗統計量を計算する．この統計量が大きければ，$X[I-Q],\ldots,X[I-1]$ がモデルに合わない．ある観測値 X_i が異常値かどうかを判断するために $\hat{u}_i/\hat{\sigma}$ と $X(i-1),\ldots X(i-p)$ のカイ二乗統計量を見ればよい．(1)の方程式の中の重みは，$\hat{u}_i$ とカイ二乗統計量の平方根の積に依存する．この重みは，WEIGHTというPROCEDUREが計算する．この重みの系列に小さい部分が含まれていると，THの古典的推定値とロバスト推定値が違う．

例：Künsch (1983) でも使用した91ヶ月の貸利率（×100）
INPUT：FILE X. DATAでROBARというプログラムの結果はFILE R. DATA（付録）にある．DIAGNOSTICSのためINPUT Y. DATAでCHECKというプログラムの結果はFILE C. DATA（付録）にある．

古典的とロバストなSGMとTHの推定値は違う．イノベーションの中では $|\hat{u}_4|$, $|\hat{u}_7|$, $|\hat{u}_{18}|$, $|\hat{u}_{19}|$, $|\hat{u}_{28}|$, $|\hat{u}_{30}|$, $|\hat{u}_{58}|$, $|\hat{u}_{77}|$ と $|\hat{u}_{88}|$ が大きい．カイ二乗統計量（$q=1$）は $i=19$, $i=29$ と $i=30$ の所で大きい．$i=18$, 19のところでは，イノベーションもカイ二乗統計量も大きく，しかも $u_{18}>0$, $u_{19}<0$ であるから $i=18$ にadditive outlierがあると判断する．データのグラフ(Künsch (1983), Fig. 2を再掲）を見てもすぐわかる．同じように，$i=28, 29$ で2つ続いてadditive outlierがあると言える．$i=4$, 7, 58, 77, 88の所でそのイノベーションのみ大きいから，ここで，イノベーション異常値があると言える．

WEIGHT w_{c_1} が小さくなっているところ（$i=4$, 18, 19, 21, 22, 28, 29, 30, 77, 88）と，上の異常値の所が完全には一致しない．w_{c_1} がカイ二乗統計量とイノベーションの両方を含んでいるために，次の場合が起る．まず，小さなカイ二乗統計量と大きなイノベーションの組み合わせである．例えば $i=7$ あるいは $i=58$ を見ればわかる．この場合，その点で不規則性があるけれど，重みは1である．一方中位の大きさのイノベーションと，中位の大きさのカイ二乗統計量の組み合せがありうる．例えば，$i=21, 22$ をみればわかる．そうすれば何も異常がなくても重みが小さくなる．このような場合，ロバスト推定量が効率を少し失う原因となる．

謝　　辞

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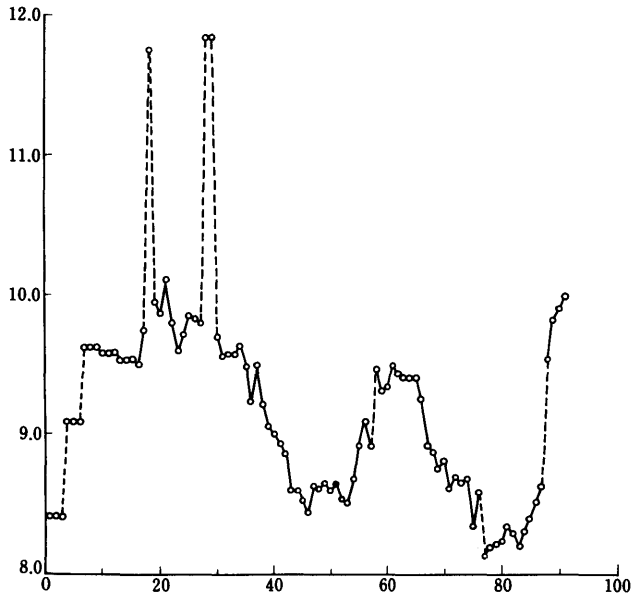

Fig. 1: 91か月の間の毎月の貸利率のデータ．点線は異常値や怪しいところを表わす．
Monthly lending rate during 91 months. The dotted lines indicate doubtful parts in the data.
（クンシュ[1], Fig. 2 の再掲）

参 考 文 献

Künsch, H. (1983). 自己回帰過程におけるロバスト推定，統計数理研究所彙報 第 **31** 巻，51～64.
Künsch, H. (1984). Infinitesimal Robustness for Autoregressive Processes, *Ann. Statist.*, **12**, 843-863.

Programs for Robust Estimation in Autoregressive Model Fitting

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Two programs for robust estimation of the parameters of autoregressive models developed in Künsch (1983) are given. The detail of the procedures can be found in the comments of the source lists.

付　録

ROBAR

```
BEGIN COMMENT THIS PROGRAMM CALCULATES OPTIMAL ROBUST ESTIMATORS          00000010
      FOR THE PARAMETERS OF AN AR-PROCESS: TH(1),..,TH(P), ETA            00000020
      AND SGM. HERE ETA=LOCATION OF THE DATA AND SGM=S.D. OF THE          00000030
      INNOVATIONS. THESE ESTIMATORS ARE DESCRIBED IN KUENSCH,             00000040
      ANN. STATIST. 12 (1984), AND KUENSCH, PROC. INST. STATIST. MATH.    00000050
      (SEPTEMBER 1983).                                                   00000060
      THE ESTIMATORS DEPEND ON TWO CONSTANTS C1 AND C2 TO BE CHOSEN       00000070
      BEFOREHAND. IF C1 INCREASES, THE ROBUSTNESS OF TH DECREASES,        00000080
      BUT THE EFFICIENCY OF TH INCREASES. THE CHOICE OF C1 DOES           00000090
      NOT AFFECT THE PROPERTIES OF ETA AND SGM. C2 REGULATES THE          00000100
      ROBUSTNESS AND EFFICIENCY OF ETA AND SGM WITHOUT AFFECTING          00000110
      TH. IN THE PROGRAMM C1 IS PUT EQUAL TO CMIN AND CMAX WHERE          00000120
      CMIN (CMAX) IS SUCH THAT THE ASYMPTOTIC EFFICIENCY OF TH            00000130
      IS 1/1.15 (1/1.05). C2 IS PUT EQUAL TO 1.0 AND 1.5.                 00000140
      THE ORDER P OF THE AR-MODEL IS VARIED BETWEEN 1 AND PMAX.           00000150
      THE PROGRAMM SHOULD WORK FOR ARBITRARY PMAX, BUT WE HAVE NO         00000160
      EXPERIENCE YET AS REGARDS NUMERICAL STABILITY AND CPU-TIME          00000170
      FOR PMAX>10.                                                        00000180
      THE ESTIMATOR IS DEFINED AS THE SOLUTION OF A NON-LINEAR SYS-       00000190
      TEM OF EQUATIONS WHICH IS SOLVED ITERATIVELY BY THE PROCE-          00000200
      DURES ARPAR AND NUISANCE. A STARTING VALUE IS CALCULATED BY         00000210
      THE PROCEDURE INITIAL FROM THE ESTIMATES FOR LOWER ORDER,           00000220
      BUT IF ALREADY AN ESIMATE FOR THE SAME ORDER, BUT DIFFERENT         00000230
      C1 OR C2 IS AVAILABLE, WE USE THIS ONE AS STARTING VALUE.           00000240
      THE PROCEDURES INITIAL, ARPAR AND NUISANCE SHOULD BE FLEXIBLE       00000250
      ENOUGH FOR MODIFICATIONS, E.G. IF ONE WANTS TO CALCULATE ONLY       00000260
      ESTIMATES FOR CERTAIN ORDERS BETWEEN 1 AND PMAX.                    00000270
      THE PROCEDURE MAXLIK IS INCLUDED FOR AN EASY COMPARISON OF          00000280
      CLASSICAL MAXIMUM LIKELIHOOD AND ROBUST ESTIMATES.                  00000290
      THE PROCEDURES LIN, INCR, MEDIAN, LOCSCL, PARCOR ARE AUXILIARY      00000350
      ONES USED AT DIFFERENT PLACES. FINALLY INDATA, OUTCOV AND           00000360
      OUTPAR ARE INPUT/OUTPUT PROCEDURES.                                 00000370
      INPUT:                                                              00000380
        N=LENGTH OF DATA VECTOR                                           00000390
        PMAX=HIGHEST ORDER OF THE MODELS TO BE FITTED                     00000400
        X=DATA VECTOR                                                     00000410
      INPUT-CHANNEL: 3                                                    00000420
      INPUT-FORMAT:                                                       00000430
        N,PMAX: 'B3ZD,B2ZD/' (MAIN PROGRAMME, ST-NO 2)                    00000440
        X: '(12(-2ZDB)/)' (PROCEDURE INDATA).                             00000450
      OUTPUT:                                                             00000460
        ML-ESTIMATES FOR ORDERS 1,..,PMAX,                                00000470
        SAMPLE COVARIANCES FOR LAGS 0,..,PMAX,                            00000480
        ROBUST ESTIMATES AND CORRESPONDING COVARIANCES                    00000490
          FOR ORDERS 1,..,PMAX, AND ROBUSTNESS CON                        00000500
          STANTS C1=CMIN,CMAX, C2=1.5,1.0,                                00000510
        INITIAL AND INTERMEDIATE ESTIMATES ARE PRINTED,                   00000520
        A WARNING IS PRINTED IF ANY OF THE ITERATIVE PRO                  00000530
        CEDURES DOES NOT CONVERGE WITHIN 10 STEPS,                        00000540
      OUTPUT-CHANNEL:1;                                                   00000550
  INTEGER I,J,K,N,PMAX,P,NI,NIT;                                          00000560
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 REAL C1,C2,CMIN,CMAX,ETA,SGM,MED;                                    00000570
 INPUT2(3,'B3ZD,B2ZD/',N,PMAX);                                       00000580
OUTPUT2(1,'B3ZD,B2ZD',N,PMAX);                                        00000590
 BEGIN ARRAY X(1:N),TH(1:PMAX),COV(0:PMAX),INIT(1:PMAX*(PMAX+5)/2);   00000600
PROCEDURE LIN(A,P); VALUE P; INTEGER P; ARRAY A;                      00000610
 COMMENT SOLUTION OF THE P-DIMENSIONAL LINEAR EQUATION                00000620
         AX+B=0. THE VECTOR B HAS TO BE PUT INTO THE (P+1)-TH         00000630
         COLUMN OF A WHICH CONTAINS THE SOLUTION X AT THE             00000640
         OUTPUT;                                                      00000650
 BEGIN REAL Z; INTEGER I,J,K;                                         00000660
   FOR K:=1 STEP 1 UNTIL P DO BEGIN                                   00000670
     Z:=1/A(K,K);                                                     00000680
     FOR J:=K+1 STEP 1 UNTIL P+1 DO                                   00000690
       A(K,J):=-A(K,J)*Z;                                             00000700
     FOR I:=1 STEP 1 UNTIL K-1, K+1 STEP 1 UNTIL P DO                 00000710
       FOR J:=K+1 STEP 1 UNTIL P+1 DO                                 00000720
         A(I,J):=A(I,J)+A(I,K)*A(K,J);                                00000730
     END K;                                                           00000740
   END LIN;                                                           00000750
 PROCEDURE INCR(X,N,M);                                               00000760
   VALUE N,M; INTEGER N,M; ARRAY X;                                   00000770
   COMMENT INCR GIVES THE SMALLEST M VALUES OF X(1),..,X(N);          00000780
   BEGIN INTEGER I,J,IM; REAL MIN;                                    00000790
     FOR I:=1 STEP 1 UNTIL M DO BEGIN                                 00000800
       MIN:=X(I);                                                     00000810
       IM:=I;                                                         00000820
       FOR J:=I+1 STEP 1 UNTIL N DO                                   00000830
         IF X(J)<MIN THEN BEGIN                                       00000840
            MIN:=X(J);                                                00000850
            IM:=J;                                                    00000860
            END THEN;                                                 00000870
       IF IM>I THEN BEGIN                                             00000880
          X(IM):=X(I);                                                00000890
          X(I):=MIN;                                                  00000900
          END THEN;                                                   00000910
       END I;                                                         00000920
   END INCR;                                                          00000930
 REAL PROCEDURE MEDIAN(X,N);                                          00000940
    VALUE N; INTEGER N; ARRAY X;                                      00000950
    COMMENT CALCULATION OF THE MEDIAN OF X(1),..,X(N);                00000960
    BEGIN INTEGER I,M; REAL Z; ARRAY HX(1:N);                         00000970
      FOR I:=1 STEP 1 UNTIL N DO                                      00000980
        HX(I):=X(I);                                                  00000990
      Z:=0.5*(N+1); M:=ENTIER(Z);                                     00001000
      INCR(HX,N,M+1);                                                 00001010
      MEDIAN:=(M+1-Z)*HX(M)+(Z-M)*HX(M+1);                            00001020
      END MEDIAN;                                                     00001030
 PROCEDURE LOCSCL(X,N,MY,S);                                          00001040
    VALUE N; INTEGER N; REAL MY,S; ARRAY X;                           00001050
    COMMENT CALCULATION OF A ROBUST ESTIMATE FOR LOCATION AND         00001060
            SCALE OF X(1),..,X(N): MY=MEDIAN OF THE X(I),             00001070
            S=0.7413*(DIFFERENCE OF QUARTILES);                       00001080
    BEGIN INTEGER I,M,M1; REAL Z,Z1; ARRAY HX(1:N);                   00001090
      FOR I:=1 STEP 1 UNTIL N DO                                      00001100
        HX(I):=X(I);                                                  00001110
      Z:=0.75*(N+1); M:=ENTIER(Z);                                    00001120
      Z1:=0.25*(N+1); M1:=ENTIER(Z1);                                 00001130
      INCR(HX,N,M+1);                                                 00001140
      S:=((M+1-Z)*HX(M)+(Z-M)*HX(M+1)-(M1+1-Z1)*HX(M1)                00001150
          -(Z1-M1)*HX(M1+1))*0.7413;                                  00001160
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      Z:=0.5*(N+1); M:=ENTIER(Z);                                    00001170
      MY:=(M+1-Z)*HX(M)+(Z-M)*HX(M+1);                               00001180
      END LOCSCL;                                                    00001190
   PROCEDURE PARCOR(P,TH,A);                                         00001200
     VALUE P; INTEGER P; ARRAY TH,A;                                 00001210
     COMMENT ADJUSTING AR-MODELS OF ORDER < P TO THE COVARAINCES      00001220
             OF THE AR(P)-MODEL WITH PARAMETERS TH(1),..,TH(P)       00001230
             AND SGM=1. IF THE STATIONARITY CONDITION DOES NOT HOLD, 00001240
             WE MODIFIY THE VALUES OF TH.                            00001250
             INPUT: TH.                                              00001260
             OUTPUT: A, WHERE A(J,J)=1/(PREDICTION VARIANCE OF THE   00001270
             MODEL OF ORDER J-1) AND A(J,K)=K-TH COEFFICIENT OF THE  00001280
             MODEL OF ORDER J-1 (K<J);                               00001290
     BEGIN REAL Z; INTEGER I,J,K;                                    00001300
     A(P+1,P+1):=1.0;                                                00001310
   RECURSION:                                                        00001320
     FOR I:=1 STEP 1 UNTIL P DO                                      00001330
       A(P+1,I):=TH(I);                                              00001340
     FOR J:=P STEP -1 UNTIL 2 DO BEGIN                               00001350
       Z:=1/(1-A(J+1,J)**2);                                         00001360
       A(J,J-1):=(A(J+1,J-1)+A(J+1,J)*A(J+1,1))*Z;                   00001370
       IF ABS(A(J,J-1))>1 THEN BEGIN                                 00001380
          FOR K:=1 STEP 1 UNTIL P DO                                 00001390
            TH(K):=0.98*TH(K);                                       00001400
          GOTO RECURSION;                                            00001410
          END THEN;                                                  00001420
       FOR K:=J-2 STEP -1 UNTIL 1 DO                                 00001430
         A(J,K):=(A(J+1,K)+A(J+1,J)*A(J+1,J-K))*Z;                   00001440
       A(J,J):=A(J+1,J+1)/Z;                                         00001450
       END J;                                                        00001460
     A(1,1):=A(2,2)*(1-A(2,1)**2);                                   00001470
     END PARCOR;                                                     00001480
PROCEDURE MAXLIK(N,P,X,COV,INIT);                                    00001490
  INTEGER N,P; ARRAY X,COV,INIT;                                     00001500
  COMMENT CALCULATION OF THE APPROXIMATED MAXIMUM LIKELIHOOD         00001510
          ESTIMATOR (SOLUTION OF THE YULE-WALKER EQUATIONS) FOR      00001520
          ALL AR-MODELS OF ORDER <=P.                                00001530
          INPUT:                                                     00001540
            N=NUMBER OF DATA                                         00001550
            P=ORDER OF THE MODEL                                     00001560
            X=DATA VECTOR.                                           00001570
          OUTPUT:                                                    00001580
            COV=SAMPLE COVARIANCES OF LAGS <=P                       00001590
            INIT=PARAMETER ESTIMATES FOR THE MODELS OF ORDER 1,..,P  00001600
            (IN THE FOLLOWING ORDER:AR-PARAMETERS, LOCATION OF DATA, 00001610
             S.D. OF INNOVATIONS);                                   00001620
   BEGIN INTEGER I,J,K,N1; REAL Z,ETA; ARRAY A(1:P+1,1:P+1);         00001630
     COMMENT CALCULATION OF SAMPLE MEAN AND SAMPLE COVARIANCES;      00001640
     ETA:=0.0;                                                       00001650
     FOR I:=1 STEP 1 UNTIL N DO                                      00001660
       ETA:=ETA+X(I);                                                00001670
     ETA:=ETA/N;                                                     00001680
     FOR I:=1 STEP 1 UNTIL N DO                                      00001690
       X(I):=X(I)-ETA;                                               00001700
     FOR K:=0 STEP 1 UNTIL P DO BEGIN                                00001710
       COV(K):=0;                                                    00001720
       FOR I:=1 STEP 1 UNTIL N-K DO                                  00001730
         COV(K):=COV(K)+X(I)*X(I+K);                                 00001740
       COV(K):=COV(K)/(N-K);                                         00001750
       END K;                                                        00001760
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    COMMENT RECURSIVE SOLUTION OF YULE-WALKER EQUATIONS FOR          00001770
            ORDERS 1 UP TO P;                                        00001780
    A(1,1):=COV(0);                                                  00001790
    N1:=1;                                                           00001800
    FOR J:=1 STEP 1 UNTIL P DO BEGIN                                 00001810
      Z:=0.0;                                                        00001820
      FOR I:=1 STEP 1 UNTIL J-1 DO                                   00001830
        Z:=Z+A(J,I)*COV(J-I);                                        00001840
      A(J+1,J):=Z:=(COV(J)-Z)/A(J,J);                                00001850
      A(J+1,J+1):=A(J,J)*(1-Z**2);                                   00001860
      FOR I:=1 STEP 1 UNTIL J-1 DO BEGIN                             00001870
        A(J+1,I):=A(J,I)-Z*A(J,J-I);                                 00001880
        INIT(N1):=A(J+1,I);                                          00001890
        N1:=N1+1;                                                    00001900
        END I;                                                       00001910
      INIT(N1):=A(J+1,J);                                            00001920
      INIT(N1+1):=ETA;                                               00001930
      INIT(N1+2):=SQRT(A(J+1,J+1));                                  00001940
      N1:=N1+3;                                                      00001950
      END J;                                                         00001960
    FOR I:=1 STEP 1 UNTIL N DO                                       00001970
      X(I):=X(I)+ETA;                                                00001980
    END MAXLIK;                                                      00001990
PROCEDURE INITIAL(N,X,Q,P,TH,COV,SGM,MED,INIT);                      00002000
  VALUE Q,P;                                                         00002010
  INTEGER N,Q,P; REAL SGM,MED; ARRAY X,TH,COV,INIT;                  00002020
  COMMENT CALCULATION OF CRUDE INITIAL ESTIMATES FOR THE ORDERS      00002030
          Q+1,..,P IF ESTIMATES FOR THE ORDER Q ARE GIVEN (THE       00002040
          CASE Q=0 IS INCLUDED). THE ESTIMATES ARE BASED ON          00002050
          MEDIAN TYPE ESTIMATES FOR THE CORRELATIONS OF LAG          00002060
          Q+1,..,P WHICH ARE MODIFIED IF POSITIVE DEFINITENESS       00002070
          DOES NOT HOLD.                                             00002080
          INPUT:                                                     00002090
          N=NUMBER OF DATA                                           00002100
          Q,P=ORDERS OF THE MODEL                                    00002110
          MED=MEDIAN OF THE DATA                                     00002120
          TH=AR-PARAMETERS OF THE MODEL OF ORDER Q                   00002130
          SGM=S.D. OF THE INNOVATIONS IN THE MODEL OF ORDER Q        00002140
          COV=COVARIANCES OF LAG<=Q IN THE MODEL OF ORDER Q          00002150
          ONLY THE VALUES COV(K)/COV(0) AND SGM**2/COV(0) ARE        00002160
          ESSENTIAL.                                                 00002170
          OUTPUT:                                                    00002180
          INIT=ESTIMATES FOR THE MODELS OF ORDER Q+1,..,P            00002190
          (IN THE FOLLOWING ORDER: AR-PARAMETERS, LOCATION OF        00002200
          DATA, S.D. OF INNOVATIONS);                                00002210
  BEGIN INTEGER N1,I,J,K; REAL U,Z;                                  00002220
        ARRAY A(1:P+1,1:P+1),Y(1:N),R(1:P);                          00002230
    COMMENT CALCULATION OF MEDIAN TYPE CORRELATIONS;                 00002240
    FOR I:=1 STEP 1 UNTIL N DO                                       00002250
      X(I):=X(I)-MED;                                                00002260
    FOR K:=1 STEP 1 UNTIL Q DO                                       00002270
      R(K):=COV(K)/COV(0);                                           00002280
    FOR K:=Q+1 STEP 1 UNTIL P DO BEGIN                               00002290
      N1:=0;                                                         00002300
      FOR I:=1 STEP 1 UNTIL N-K DO                                   00002310
        IF ABS(X(I))>0.00001 THEN BEGIN                              00002320
           N1:=N1+1;                                                 00002330
           Y(N1):=X(I+K)/X(I);                                       00002340
           END THEN;                                                 00002350
      R(K):=MEDIAN(Y,N1);                                            00002360
```

```
      END K;                                                          00002370
    COMMENT RECURSIVE SOLUTION OF THE YULE-WALKER EQUATIONS FOR       00002380
            ORDERS Q+1 UP TO P;                                       00002390
    FOR I:=1 STEP 1 UNTIL Q DO                                        00002400
      A(Q+1,I):=TH(I);                                                00002410
    A(Q+1,Q+1):=SGM**2/COV(0);                                        00002420
    FOR J:=Q+1 STEP 1 UNTIL P DO BEGIN                                00002430
      Z:=0.0;                                                         00002440
      FOR I:=1 STEP 1 UNTIL J-1 DO                                    00002450
        Z:=Z+A(J,I)*R(J-I);                                           00002460
      IF A(J,J)<0.5 THEN U:=1.7*A(J,J)**2                             00002470
         ELSE U:=A(J,J)*(A(J,J)*0.2+0.75);                            00002480
      IF R(J)>Z+U THEN R(J):=Z+U;                                     00002490
      IF R(J)<Z-U THEN R(J):=Z-U;                                     00002500
      A(J+1,J):=Z:=(R(J)-Z)/A(J,J);                                   00002510
      A(J+1,J+1):=A(J,J)*(1-Z**2);                                    00002520
      FOR I:=1 STEP 1 UNTIL J-1 DO                                    00002530
        A(J+1,I):=A(J,I)-Z*A(J,J-I);                                  00002540
      END J;                                                          00002550
    COMMENT ROBUST LOCATION AND SCALE ESTIMATES FOR THE INNOVATIONS;   00002560
    FOR I:=1 STEP 1 UNTIL N DO                                        00002570
      X(I):=X(I)+MED;                                                 00002580
    N1:=1;                                                            00002590
    FOR J:=Q+2 STEP 1 UNTIL P+1 DO BEGIN                              00002600
      Z:=1.0;                                                         00002610
      FOR I:=1 STEP 1 UNTIL J-1 DO BEGIN                              00002620
        INIT(N1):=A(J,I);                                             00002630
        N1:=N1+1;                                                     00002640
        Z:=Z-A(J,I);                                                  00002650
        END I;                                                        00002660
      FOR I:=1 STEP 1 UNTIL N-J+1 DO BEGIN                            00002670
        Y(I):=X(I+J-1);                                               00002680
        FOR K:=1 STEP 1 UNTIL J-1 DO                                  00002690
          Y(I):=Y(I)-A(J,K)*X(I+J-1-K);                               00002700
        END I;                                                        00002710
      LOCSCL(Y,N-J+1,INIT(N1),INIT(N1+1));                            00002720
      INIT(N1):=INIT(N1)/Z;                                           00002730
      N1:=N1+2;                                                       00002740
      END J;                                                          00002750
      X(I):=X(I)+INIT(1);                                             00002760
    END INITIAL;                                                      00002770
PROCEDURE ARPAR(N,P,X,ETA,SGM,C1,TH,COV,NI);                          00002780
  INTEGER N,P,NI; REAL ETA,SGM,C1; ARRAY X,TH,COV;                    00002790
  COMMENT CALCULATION OF A ROBUST ESTIMATE FOR THE AUTOREG            00002800
          RESSIVE PARAMETER TH(1),..,TH(P) FOR KNOWN VALUES           00002810
          OF ETA AND SGM. THE ESTIMATES ARE SOLUTIONS OF "LINEAR"     00002820
          EQUATIONS SIMILAR TO YULE-WALKER, BUT WEIGHTED SAMPLE CO    00002830
          VARIANCES ARE USED WHERE THE WEIGHTS DEPEND ON THE UNKNOWN  00002840
          TH AND THE DATA.                                            00002850
          INPUT:                                                      00002860
            N=NUMBER OF DATA                                          00002870
            P=ORDER OF THE MODEL                                      00002880
            X=DATA VECTOR                                             00002890
            ETA=MEAN OF THE DATA                                      00002900
            SGM=S.D. OF THE INNOVATIONS                               00002910
            C1=TRUNCATION POINT OF THE ESTIMATOR                      00002920
            TH=STARTING ESTIMATE                                      00002930
          OUTPUT:                                                     00002940
            TH=FINAL ESTIMATE                                         00002950
            COV=COVARIANCES OF THE FITTED MODEL                       00002960
```

```
          NI=NUMBER OF ITERATIONS NEEDED;                                    00002970
   BEGIN INTEGER I,J,K; REAL C,W,U,Z;                                        00002980
         ARRAY Y(1:N),DIFF(1:P),A,R(1:P+1,1:P+2);                            00002990
REAL PROCEDURE WEIGHT(I,P,TH);                                               00003000
  VALUE I,P; INTEGER I,P; ARRAY TH;                                          00003010
  COMMENT CALCULATION OF THE WEIGHT OF THE P-TUPLE                           00003020
    Y(I),Y(I+1),..,Y(I+P) IN THE MODEL WITH PARAMETERS TH;                   00003030
  BEGIN REAL U,V,Z;                                                          00003040
    U:=Y(I+P);                                                               00003050
    FOR J:=1 STEP 1 UNTIL P DO                                               00003060
      U:=U-TH(J)*Y(I+P-J);                                                   00003070
    Z:=0;                                                                    00003080
    FOR J:=0 STEP 1 UNTIL P-1 DO BEGIN                                       00003090
      V:=Y(I+J);                                                             00003100
      FOR K:=1 STEP 1 UNTIL J DO                                             00003110
        V:=V-A(J+1,K)*Y(I+J-K);                                              00003120
      Z:=Z+A(J+1,J+1)*V**2;                                                  00003130
      END J;                                                                 00003140
    WEIGHT:=SQRT(Z)*ABS(U);                                                  00003150
    END WEIGHT I;                                                            00003160
     FOR I:=1 STEP 1 UNTIL N DO                                              00003170
       Y(I):=X(I)-ETA;                                                       00003180
     NI:=0;                                                                  00003190
   ITER:                                                                     00003200
     NI:=NI+1;                                                               00003210
     IF NI=11 THEN GOTO ENDE;                                                00003220
     COMMENT CALCULATION OF COVARIANCES AND AR-MODELS OF SMALLER              00003230
             ORDER BASED ON THE OLD TH;                                      00003240
     PARCOR(P,TH,A);                                                         00003250
     COV(0):=SGM**2/A(1,1);                                                  00003260
     FOR K:=1 STEP 1 UNTIL P DO BEGIN                                        00003270
       COV(K):=0.0;                                                          00003280
       FOR I:=1 STEP 1 UNTIL K DO                                            00003290
         COV(K):=COV(K)+A(K+1,I)*COV(K-I);                                   00003300
       END K;                                                                00003310
     COMMENT CALCULATION OF WEIGHTED SAMPLE COVARIANCES AND SOLUTION         00003320
             OF THE LINEAR EQUATION;                                         00003330
     C:=C1*SGM**2;                                                           00003340
     FOR J:=1 STEP 1 UNTIL P DO                                              00003350
       FOR K:=J STEP 1 UNTIL P+1 DO                                          00003360
         R(J,K):=0;                                                          00003370
     FOR I:=1 STEP 1 UNTIL N-P DO BEGIN                                      00003380
       W:=WEIGHT(I,P,TH);                                                    00003390
       IF W<C THEN W:=1 ELSE W:=C/W;                                         00003400
       FOR J:=1 STEP 1 UNTIL P DO                                            00003410
         FOR K:=J STEP 1 UNTIL P+1 DO                                        00003420
           R(J,K):=R(J,K)+W*Y(I+J-1)*Y(I+K-1);                               00003430
       END I;                                                                00003440
     FOR J:=1 STEP 1 UNTIL P DO BEGIN                                        00003450
       FOR K:=1 STEP 1 UNTIL J-1 DO                                          00003460
         R(J,K):=R(K,J);                                                     00003470
       R(J,P+1):=-R(J,P+1);                                                  00003480
       END J;                                                                00003490
     LIN(R,P);                                                               00003500
     COMMENT MEASURING THE DIFFERENCE (NEW TH - OLD TH) IN THE METRIC         00003510
             OF THE INFORMATION MATRIX. IF IT IS SMALL, THE ITERATION         00003520
             IS STOPPED;                                                     00003530
     FOR I:=1 STEP 1 UNTIL P DO BEGIN                                        00003540
       DIFF(I):=TH(I)-R(P-I+1,P+1);                                          00003550
       TH(I):=R(P-I+1,P+1);                                                  00003560
```

```
      END I;                                                          00003570
    Z:=0.0;                                                           00003580
    FOR K:=0 STEP 1 UNTIL P-1 DO BEGIN                                00003590
      U:=0.0;                                                         00003600
      FOR I:=1 STEP 1 UNTIL P-K DO                                    00003610
        U:=U+DIFF(I)*DIFF(I+K);                                       00003620
      IF K>0 THEN U:=2*U;                                             00003630
      Z:=Z+COV(K)*U;                                                  00003640
      END K;                                                          00003650
    IF Z>0.002*P*SGM**2/N THEN GOTO ITER;                             00003660
  ENDE:                                                               00003670
    END ARPAR;                                                        00003680
PROCEDURE NUISANCE(N,P,X,TH,C1,ETA,SGM,NI);                           00003690
  INTEGER N,P,NI; REAL C1,ETA,SGM; ARRAY X,TH;                        00003700
  COMMENT CALCULATION OF A ROBUST ESTIMATE FOR THE NUISANCE           00003710
          PARAMETERS ETA AND SGM FOR KNOWN VALUES OF THE              00003720
          AUTOREGRESSIVE PARAMETER TH. CONSIDERING THE INNOVATIONS,   00003730
          THIS BECOMES AN I.I.D. LOCATION-SCALE PROBLEM. THE METHOD   00003740
          USED IS VARIANT 4 OF HUBER, ROBUST STATISTICS, P.147.       00003750
          INPUT:                                                      00003760
            N=NUMBER OF DATA                                          00003770
            P=ORDER OF THE MODEL                                      00003780
            X=DATA VECTOR                                             00003790
            TH=AUTOREGRESSIVE PARAMETERS                              00003800
            C1=TRUNCATION POINT OF THE ESTIMATOR                      00003810
               (C1 HAS TO BE ONE OF THE VALUES 1.5,1.0,0.5)           00003820
            ETA,SGM=STARTING ESTIMATES                                00003830
          OUTPUT:                                                     00003840
            ETA,SGM=FINAL ESTIMATES                                   00003850
            NI=NUMBER OF ITERATIONS NEEDED;                           00003860
  BEGIN                                                               00003870
    INTEGER N1,N2,N3,I,J; REAL U,U1,U2,U3,C,C3,EPS;                   00003880
    INTEGER ARRAY CASE(1:N); REAL ARRAY Y(1:N);                       00003890
    COMMENT CALCULATION OF INNOVATIONS;                               00003900
    FOR I:=1 STEP 1 UNTIL N-P DO BEGIN                                00003910
      Y(I):=X(I+P);                                                   00003920
      FOR J:=1 STEP 1 UNTIL P DO                                      00003930
        Y(I):=Y(I)-TH(J)*X(I+P-J);                                    00003940
      END I;                                                          00003950
    U:=1.0;                                                           00003960
    FOR I:=1 STEP 1 UNTIL P DO                                        00003970
      U:=U-TH(I);                                                     00003980
    ETA:=ETA*U;                                                       00003990
    IF C1=0.5 THEN C3:=0.0926;                                        00004000
    IF C1=1.0 THEN C3:=0.516;                                         00004010
    IF C1=1.5 THEN C3:=0.7785;                                        00004020
    NI:=0;                                                            00004030
  ITER:                                                               00004040
    NI:=NI+1;                                                         00004050
    IF NI=11 THEN GOTO ENDE;                                          00004060
    COMMENT PARTITIONING OF THE INNOVATIONS ACCORDING TO THE OLD      00004070
            ETA AND SGM;                                              00004080
    C:=C1*SGM;                                                        00004090
    N1:=N2:=N3:=0; U:=0.0;                                            00004100
    FOR I:=1 STEP 1 UNTIL N-P DO BEGIN                                00004110
      IF Y(I)<ETA-C THEN CASE(I):=1 ELSE                              00004120
         IF Y(I)<ETA+C THEN CASE(I):=2 ELSE CASE(I):=3;               00004130
      IF CASE(I)=1 THEN N1:=N1+1 ELSE                                 00004140
         IF CASE(I)=2 THEN N2:=N2+1 ELSE N3:=N3+1;                    00004150
      IF CASE(I)=2 THEN U:=U+Y(I);                                    00004160
```

```
    END I;                                                              00004170
   U:=U/N2; U1:=0.0;                                                    00004180
   COMMENT CALCULATION OF NEW VALUES OF ETA AND SGM;                    00004190
   FOR I:=1 STEP 1 UNTIL N-P DO                                         00004200
     IF CASE(I)=2 THEN U1:=U1+(Y(I)-U)**2;                              00004210
   U2:=SGM;                                                             00004220
   SGM:=SQRT(U1/((N-P-1)*C3-(N1+N3+(N3-N1)**2/N2)*C1**2));              00004230
   U3:=ETA; ETA:=U+SGM*C1*(N3-N1)/N2;                                   00004240
   EPS:=0.05*SGM/SQRT(N);                                               00004250
   IF ABS(SGM-U2)<0.5*EPS & ABS(ETA-U3)<EPS THEN GOTO ENDE;             00004260
   GOTO ITER;                                                           00004270
 ENDE:                                                                  00004280
   U:=1.0;                                                              00004290
   FOR I:=1 STEP 1 UNTIL P DO                                           00004300
     U:=U-TH(I);                                                        00004310
   ETA:=ETA/U;                                                          00004320
   END NUISANCE;                                                        00004330
PROCEDURE OUTPAR(P,I);                                                  00004820
  INTEGER P,I;                                                          00004830
  COMMENT PRINTING OF THE PARAMETER ESTIMATES TH,ETA AND SGM            00004840
          IN A MODEL OF ORDER P. I IS USED TO DISTINGUISH DIF           00004850
          FERENT CASES FOR THE TITLE OF THE OUTPUT;                     00004860
  BEGIN INTEGER J;                                                      00004870
    IF I=1 THEN OUTPUT0(1,'/"APPROXIMATED MAX. LIK. ESTIMATES"');       00004880
    IF I=2 THEN OUTPUT0(1,'"CRUDE INITIAL ROBUST ESTIMATES"');          00004890
    IF I=3 THEN BEGIN                                                   00004900
       OUTPUT0(1,'/"FINAL ESTIMATES WITH ROBUSTNESS CONSTANTS"');       00004910
       OUTPUT2(1,'" C1="ZZ.DD,", C2="ZZ.DD',C1,C2);                     00004920
       END THEN;                                                        00004930
    IF I=4 THEN                                                         00004940
   OUTPUT0(1,'"INTERMEDIATE ESTIMATES IN THE RECURSIVE CALCULATION"/');00004950
    IF I<4 THEN OUTPUT1(1,'" FOR THE ORDER"3Z/',P);                     00004960
    OUTPUT0(1,'" AR-PARAMETERS"');                                      00004970
    FOR J:=1 STEP 1 UNTIL P DO                                          00004980
      OUTPUT1(1,'-2Z.3D',TH(J));                                        00004990
    OUTPUT1(1,'/" LOCATION OF DATA:"-5Z.3D',ETA);                       00005000
    OUTPUT1(1,'3B"S.D. OF INNOVATIONS:"5Z.3D/',SGM);                    00005010
    END OUTPAR;                                                         00005020
PROCEDURE OUTCOV(P);                                                    00005030
  INTEGER P;                                                            00005040
  COMMENT PRINTING OF COVARIANCES COV(J), J=0,..P;                      00005050
    BEGIN INTEGER J;                                                    00005060
    OUTPUT1(1,'" CORRESPONDING COVARIANCES FOR LAGS 0,..,"3Z/',P);      00005070
    FOR J:=0 STEP 1 UNTIL P DO                                          00005080
      OUTPUT1(1,'-8Z.6D',COV(J));                                       00005090
    OUTPUT0(1,'//');                                                    00005100
    END OUTCOV;                                                         00005110
PROCEDURE INDATA(X,N);                                                  00005120
  INTEGER N; ARRAY X;                                                   00005130
  COMMENT READING OF DATA AND STORING IN THE ARRAY X. N=NUMBER OF DATA;00005140
  BEGIN INTEGER I,J;                                                    00005150
    FOR I:=1 STEP 14 UNTIL N DO BEGIN                                   00005160
      FOR J:=I STEP 1 UNTIL MIN(I+13,N) DO BEGIN                        00005170
        INPUT1(3,'-2ZDD',X(J));                                         00005180
        OUTPUT2(1,'3ZD,-3ZDB',J,X(J));                                  00005190
        END J;                                                          00005200
       INPUT0(3,'/');                                                   00005210
      END I;                                                            00005220
    END INDATA;                                                         00005230
INDATA(X,N);                                                            00005240
```

```
COMMENT CALCULATION AND OUTPUT OF MAX.LIK.ESTIMATES;                    00005250
MAXLIK(N,PMAX,X,COV,INIT);                                              00005260
K:=0;                                                                   00005270
FOR P:=1 STEP 1 UNTIL PMAX DO BEGIN                                     00005280
  FOR J:=1 STEP 1 UNTIL P DO                                            00005290
    TH(J):=INIT(K+J);                                                   00005300
  ETA:=INIT(K+P+1);                                                     00005310
  SGM:=INIT(K+P+2);                                                     00005320
  OUTPAR(P,1);                                                          00005330
  K:=K+P+2;                                                             00005340
  END P;                                                                00005350
OUTCOV(PMAX);                                                           00005360
COMMENT CALCULATION OF INITIAL ROBUST ESTIMATES;                        00005370
MED:=MEDIAN(X,N);                                                       00005380
SGM:=COV(0):=1.0;                                                       00005390
FOR P:=1 STEP 1 UNTIL PMAX DO BEGIN                                     00005400
  INITIAL(N,X,P-1,P,TH,COV,SGM,MED,INIT);                               00005410
  FOR J:=1 STEP 1 UNTIL P DO                                            00005420
    TH(J):=INIT(J);                                                     00005430
  ETA:=INIT(P+1);                                                       00005440
  SGM:=INIT(P+2);                                                       00005450
  OUTPAR(P,2);                                                          00005460
  COMMENT CALCULATION OF CMIN AND CMAX;                                 00005470
  IF P<5 THEN CMIN:=1.3+P*0.2                                           00005480
     ELSE IF P<9 THEN CMIN:=1.5+P*0.15                                  00005490
       ELSE CMIN:=0.34+SQRT(P)*0.82;                                    00005500
  IF P<3 THEN CMAX:=2.1+P*0.4                                           00005510
     ELSE IF P<5 THEN CMAX:=2.3+P*0.3                                   00005520
       ELSE IF P<7 THEN CMAX:=2.5+P*0.25                                00005530
         ELSE IF P<10 THEN CMAX:=2.8+P*0.2                              00005540
           ELSE CMAX:=0.52+SQRT(P)*1.37;                                00005550
COMMENT ITERATIVE CALCULATION OF FINAL ROBUST ESTIMATE FOR              00005560
        DIFFERENT VALUES OF C1 AND C2. ALWAYS ONE OF THE                00005570
        TWO CONSTANTS C1,C2 IS KEPT UNCHANGED. IF C1 CHANGES            00005580
        (I=3), WE START WITH ARPAR, IF C2 CHANGES (I=2,4) OR            00005590
        AT THE BEGINNING (I=1) WE START WITH NUISANCE;                  00005600
FOR I:=1 STEP 1 UNTIL 4 DO BEGIN                                        00005610
  IF I<3 THEN C1:=CMAX ELSE C1:=CMIN;                                   00005620
  IF REM(I,3)=1 THEN C2:=1.5 ELSE C2:=1.0;                              00005630
  IF I=3 THEN BEGIN                                                     00005640
     ARPAR(N,P,X,ETA,SGM,C1,TH,COV,NI);                                 00005650
     IF NI=11 THEN                                                      00005660
        OUTPUTO(1,'"ITERATION FOR TH HAS NOT YET CONVERGED"/');         00005670
     END THEN;                                                          00005680
NUISANCE(N,P,X,TH,C2,ETA,SGM,NI);                                       00005690
IF NI=11 THEN                                                           00005700
   OUTPUTO(1,'"ITERATION FOR ETA/SGM HAS NOT YET CONVERGED"/');         00005710
IF I=3 THEN OUTPAR(P,4);                                                00005720
ARPAR(N,P,X,ETA,SGM,C1,TH,COV,NI);                                      00005730
IF NI=11 THEN                                                           00005740
   OUTPUTO(1,'"ITERATION FOR TH HAS NOT YET CONVERGED"/');              00005750
IF I=3 & NI=1 THEN GOTO STOP;                                           00005760
IF REM(I,3)>0 THEN OUTPAR(P,4);                                         00005770
NIT:=1;                                                                 00005780
FOR NIT:=NIT+1 WHILE NIT<11 DO BEGIN                                    00005790
  NUISANCE(N,P,X,TH,C2,ETA,SGM,NI);                                     00005800
  IF NI=1 THEN GOTO STOP;                                               00005810
  IF NI=11 THEN                                                         00005820
  OUTPUTO(1,'"ITERATION FOR ETA/SGM HAS NOT YET CONVERGED"/');          00005830
  IF I=3 THEN OUTPAR(P,5);                                              00005840
```

```
  ARPAR(N,P,X,ETA,SGM,C1,TH,COV,NI);                                    00005850
  IF NI=1 THEN GOTO STOP;                                               00005860
  IF NI=11 THEN                                                         00005870
  OUTPUTO(1,'"ITERATION FOR TH HAS NOT YET CONVERGED"/');               00005880
  IF REM(I,3)>0 THEN OUTPAR(P,5);                                       00005890
  END WHILE;                                                            00005900
STOP:                                                                   00005910
IF NIT=11 THEN                                                          00005920
   OUTPUTO(1,'/"OVERALL ITERATION HAS NOT YET CONVERGED"');              00005930
OUTPAR(P,3);                                                            00005940
OUTCOV(P);                                                              00005950
END I;                                                                  00005960
END P;                                                                  00005970
END; END                                                                00005980
```

R. DATA (ROBAR出力例)

```
 91   2   1  842   2  842   3  842   4  910   5  910   6  910   7  963
  8 963   9  963  10  959  11  959  12  959  13  954  14  954  15  954
 16 951  17  975  18 1175  19  995  20  987  21 1011  22  980  23  960
 24 972  25  985  26  982  27  980  28 1184  29 1184  30  970  31  956
 32 958  33  958  34  964  35  949  36  924  37  950  38  922  39  906
 40 901  41  894  42  887  43  861  44  861  45  853  46  845  47  864
 48 862  49  866  50  861  51  865  52  854  53  852  54  869  55  893
 56 910  57  893  58  947  59  932  60  935  61  950  62  945  63  942
 64 942  65  942  66  927  67  893  68  888  69  877  70  882  71  862
 72 870  73  866  74  869  75  835  76  859  77  814  78  820  79  822
 80 824  81  834  82  830  83  821  84  831  85  840  86  852  87  864
 88 955  89  982  90  991  91 1000
```

```
 APPROXIMATED MAX. LIK. ESTIMATES FOR THE ORDER  1
  AR-PARAMETERS    .789
  LOCATION OF DATA:    918.637   S.D. OF INNOVATIONS:    44.263

 APPROXIMATED MAX. LIK. ESTIMATES FOR THE ORDER  2
  AR-PARAMETERS    .766    .030
  LOCATION OF DATA:    918.637   S.D. OF INNOVATIONS:    44.243
  CORRESPONDING COVARIANCES FOR LAGS 0,..,  2
      5193.519530      4098.453120      3292.966800

 CRUDE INITIAL ROBUST ESTIMATES FOR THE ORDER  1
  AR-PARAMETERS    .950
  LOCATION OF DATA:    925.503   S.D. OF INNOVATIONS:    12.686
 INTERMEDIATE ESTIMATES IN THE RECURSIVE CALCULATION
  AR-PARAMETERS    .958
  LOCATION OF DATA:    916.452   S.D. OF INNOVATIONS:    15.498

 FINAL ESTIMATES WITH ROBUSTNESS CONSTANTS C1= 2.50, C2= 1.50 FOR THE ORDER  1
  AR-PARAMETERS    .958
  LOCATION OF DATA:    917.961   S.D. OF INNOVATIONS:    15.436
  CORRESPONDING COVARIANCES FOR LAGS 0,..,  1
      2905.479490      2783.798830

 INTERMEDIATE ESTIMATES IN THE RECURSIVE CALCULATION
  AR-PARAMETERS    .959
  LOCATION OF DATA:    911.029   S.D. OF INNOVATIONS:    13.358
```

```
FINAL ESTIMATES WITH ROBUSTNESS CONSTANTS C1= 2.50, C2= 1.00 FOR THE ORDER  1
 AR-PARAMETERS    .959
 LOCATION OF DATA:    911.048    S.D. OF INNOVATIONS:    13.334
 CORRESPONDING COVARIANCES FOR LAGS 0,..,  1
     2185.687740      2094.563720

INTERMEDIATE ESTIMATES IN THE RECURSIVE CALCULATION
 AR-PARAMETERS    .959
 LOCATION OF DATA:    911.047    S.D. OF INNOVATIONS:    13.333

FINAL ESTIMATES WITH ROBUSTNESS CONSTANTS C1= 1.50, C2= 1.00 FOR THE ORDER  1
 AR-PARAMETERS    .959
 LOCATION OF DATA:    911.047    S.D. OF INNOVATIONS:    13.333
 CORRESPONDING COVARIANCES FOR LAGS 0,..,  1
     2214.107180      2123.360110

INTERMEDIATE ESTIMATES IN THE RECURSIVE CALCULATION
 AR-PARAMETERS    .959
 LOCATION OF DATA:    918.166    S.D. OF INNOVATIONS:    15.432

FINAL ESTIMATES WITH ROBUSTNESS CONSTANTS C1= 1.50, C2= 1.50 FOR THE ORDER  1
 AR-PARAMETERS    .959
 LOCATION OF DATA:    918.253    S.D. OF INNOVATIONS:    15.430
 CORRESPONDING COVARIANCES FOR LAGS 0,..,  1
     2966.206050      2844.646240

CRUDE INITIAL ROBUST ESTIMATES FOR THE ORDER  2
 AR-PARAMETERS    .828    .136
 LOCATION OF DATA:    930.333    S.D. OF INNOVATIONS:    15.475
INTERMEDIATE ESTIMATES IN THE RECURSIVE CALCULATION
 AR-PARAMETERS    .967   -.019
 LOCATION OF DATA:    916.076    S.D. OF INNOVATIONS:    17.645
 AR-PARAMETERS    .976   -.022
 LOCATION OF DATA:    918.049    S.D. OF INNOVATIONS:    15.493

FINAL ESTIMATES WITH ROBUSTNESS CONSTANTS C1= 2.90, C2= 1.50 FOR THE ORDER  2
 AR-PARAMETERS    .977   -.023
 LOCATION OF DATA:    919.224    S.D. OF INNOVATIONS:    15.433
 CORRESPONDING COVARIANCES FOR LAGS 0,..,  2
     2685.132810      2563.224850      2441.580810

INTERMEDIATE ESTIMATES IN THE RECURSIVE CALCULATION
 AR-PARAMETERS    .991   -.030
 LOCATION OF DATA:    913.316    S.D. OF INNOVATIONS:    13.111

FINAL ESTIMATES WITH ROBUSTNESS CONSTANTS C1= 2.90, C2= 1.00 FOR THE ORDER  2
 AR-PARAMETERS    .993   -.032
 LOCATION OF DATA:    914.292    S.D. OF INNOVATIONS:    12.732
 CORRESPONDING COVARIANCES FOR LAGS 0,..,  2
     2152.637210      2069.927000      1985.484860

INTERMEDIATE ESTIMATES IN THE RECURSIVE CALCULATION
 AR-PARAMETERS   1.002   -.031
 LOCATION OF DATA:    916.062    S.D. OF INNOVATIONS:    12.468

FINAL ESTIMATES WITH ROBUSTNESS CONSTANTS C1= 1.70, C2= 1.00 FOR THE ORDER  2
 AR-PARAMETERS   1.003   -.031
 LOCATION OF DATA:    916.062    S.D. OF INNOVATIONS:    12.468
 CORRESPONDING COVARIANCES FOR LAGS 0,..,  2
     2802.237550      2723.333250      2641.896240
```

```
INTERMEDIATE ESTIMATES IN THE RECURSIVE CALCULATION
 AR-PARAMETERS  1.001  -.035
 LOCATION OF DATA:   926.157   S.D. OF INNOVATIONS:   15.411

FINAL ESTIMATES WITH ROBUSTNESS CONSTANTS C1= 1.70, C2= 1.50 FOR THE ORDER  2
 AR-PARAMETERS  1.001  -.035
 LOCATION OF DATA:   923.557   S.D. OF INNOVATIONS:   15.366
 CORRESPONDING COVARIANCES FOR LAGS 0,..,  2
     3644.482670     3524.294920     3399.832030



>>.
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CHECK

```
   BEGIN COMMENT THIS POGRAMME CONTAINS THREE PRROCEDURES WHICH HELP  00000010
                 TO DETECT OUTLIERS AND OTHER IRREGULARITIES IN       00000020
                 TIME SERIES. IT IS ASSUMED THAT AN AR(P)-MODEL HAS   00000030
                 BEEN FITTED BY THE ROBUST ESTIMATION PROGRAMME       00000040
                 ROBAR.ALG. THE THREE PROCEDURES DO THE FOLLOWING:    00000050
                 INNOV LISTS ALL STANDARDIZED INNOVATIONS. LARGE      00000060
                       VALUES ARE RESPONSIBLE FOR DIFFERENCES BETWEEN 00000070
                       THE ROBUST AND THE NON-ROBUST ESTIMATES FOR    00000080
                       ETA:=E(X(I)) AND SGM:=S.D. OF THE INNOVATIONS. 00000090
                 CHISQ DETECTS THOSE Q-TUPLES OF CONSECUTIVE OBSER-   00000100
                       VATIONS WHICH DO NOT FIT TO THE MODEL.         00000110
                 WEIGHT GIVES THE DATA DEPENDENT WEIGHTS USED IN THE  00000120
                        ROBUST ESTIMATION. A WEIGHT << 1 MEANS THAT   00000130
                        THE CORRESPONDING (P+1)-TUPLE IS RESPONSIBLE  00000140
                        FOR A DIFFERENCE BETWEEN THE ROBUST AND THE   00000150
                       NON-ROBUST ESTIMATES FOR THE AUTOREGRESSIVE    00000160
                       PARAMETER. THE WEIGHTS ARE FORMED WITH THE     00000170
                       INNOVATIONS AND THE CHI-SQUARE STATISTIC FOR   00000180
                       THE PRECEEDING P VALUES, SO FOR A MORE DE-     00000190
                       TAILLED ANALYSIS ONE HAS TO LOOK AT THE OUT-   00000200
                       PUTS OF INNOV AND CHISQ.                       00000210
                 INPUT: N=NUMBER OF DATA                              00000220
                        P=ORDER OF THE MODEL                          00000230
                        TH=AR-PARAMETERS                              00000240
                        ETA=LOCATION OF THE DATA                      00000250
                        SGM=S.D. OF THE INNOVATIONS                   00000260
                        C=ROBUSTNESS CONSTANT USED FOR TH             00000270
                        X=DATA VECTOR                                 00000280
                        ARE READ IN THIS ORDER FROM THE FILE Y.DAT.   00000290
                  OUTPUT: THE OUTPUT FROM THE PROCEDURES INNOV, CHISQ 00000300
                          AND WEIGHT IS PRINTED ON THE FILE C.DAT;    00000310
    INTEGER N,I,J,P; REAL C,ETA,SGM;                                  00000320
    INPUT2(3,'B3ZD,B2ZD/',N,P);                                       00000330
   OUTPUT2(1,'B3ZD,B2ZD',N,P);                                        00000340
   OUTPUT0(1,'//');                                                   00000350
     BEGIN ARRAY X(1:N), TH(1:P);                                     00000360
     PROCEDURE PARCOR(P,TH,A);                                        00000370
       VALUE P; INTEGER P; ARRAY TH,A;                                00000380
       COMMENT ADJUSTING AR-MODELS OF ORDER < P TO THE COVARIANCES    00000390
               OF THE AR(P)-MODEL WITH PARAMETERS TH(1),..,TH(P)      00000400
               AND SGM=1.                                             00000410
```

```
            INPUT: TH.                                                  00000420
            OUTPUT: A, WHERE A(J,J)=1/(PREDICTION VARIANCE OF THE00000430
            MODEL OF ORDER J-1) AND A(J,K)=K-TH COEFFICIENT OF TH00000440
            MODEL OF ORDER J-1 (K<J);                                   00000450
    BEGIN REAL Z; INTEGER I,J,K;                                        00000460
    A(P+1,P+1):=1.0;                                                    00000470
    FOR I:=1 STEP 1 UNTIL P DO                                          00000480
      A(P+1,I):=TH(I);                                                  00000490
    FOR J:=P STEP -1 UNTIL 2 DO BEGIN                                   00000500
      Z:=1/(1-A(J+1,J)**2);                                             00000510
      A(J,J-1):=(A(J+1,J-1)+A(J+1,J)*A(J+1,1))*Z;                       00000520
      FOR K:=J-2 STEP -1 UNTIL 1 DO                                     00000530
        A(J,K):=(A(J+1,K)+A(J+1,J)*A(J+1,J-K))*Z;                       00000540
      A(J,J):=A(J+1,J+1)/Z;                                             00000550
      END J;                                                            00000560
    A(1,1):=A(2,2)*(1-A(2,1)**2);                                       00000570
    END PARCOR;                                                         00000580
PROCEDURE INNOV(N,P,X,TH);                                              00000590
  INTEGER N,P; ARRAY X,TH;                                              00000600
  COMMENT CALCULATION OF THE INNOVATIONS FOR THE AR(P)-MODEL            00000610
          WITH PARAMETERS TH AND ETA=0.                                 00000620
          INPUT:                                                        00000630
          N=NUMBER OF DATA                                              00000640
          P=ORDER OF THE MODEL                                          00000650
          X=DATA VECTOR                                                 00000660
          TH=AR-PARAMETER                                               00000670
          THE RESULTS ARE PRINTED DURING THE PROCEDURE,                 00000680
          FOR I=1,2,..P-1 EMPTY SPACES ARE PRINTED;                     00000690
  BEGIN INTEGER I,J; REAL U;                                            00000700
    FOR I:=1 STEP 1 UNTIL P DO                                          00000710
        SPACE(7);                                                       00000720
    FOR I:=P+1 STEP 1 UNTIL N DO BEGIN                                  00000730
        U:=X(I);                                                        00000740
        FOR J:=1 STEP 1 UNTIL P DO                                      00000750
          U:=U-TH(J)*X(I-J);                                            00000760
        OUTPUT1(1,'-7Z.3D',U);                                          00000770
        IF REM(I,5)=0 THEN OUTPUT0(1,'/');                              00000780
        END I;                                                          00000790
    OUTPUT0(1,'//');                                                    00000800
    END INNOV;                                                          00000810
PROCEDURE CHISQ(N,P,Q,X,TH);                                            00000820
  INTEGER N,P,Q; ARRAY X,TH;                                            00000830
  COMMENT THE CHI-SQUARE STATISTICS OF ALL Q-TUPLES (Q<=P) X(I-Q),00000840
          ...,X(I-1) ARE CALCULATED BASED ON AN AR(P)-MODEL             00000850
          WITH PARAMETERS TH, ETA=0, SGM=1.                             00000860
          INPUT:                                                        00000870
            N=NUMBER OF DATA                                            00000880
            X=DATA VECTOR                                               00000890
            P=ORDER OF THE MODEL                                        00000900
            Q=NUMBER OF CONSECUTIVE DATA TO INVESTIGATE                 00000910
            TH,ETA,SGM=PARAMETERS OF THE MODEL                          00000920
          ALL RESULTS ARE PRINTED DURING THE PROCEDURE,                 00000930
          FOR I=1,..,Q EMPTY SPACES ARE PRINTED;                        00000940
  BEGIN INTEGER I,J,K; REAL V,Z;                                        00000950
        ARRAY A(1:P+1,1:P+2);                                           00000960
      PARCOR(P,TH,A);                                                   00000970
      FOR I:=1 STEP 1 UNTIL Q DO SPACE(7);                              00000980
      FOR I:=Q+1 STEP 1 UNTIL N DO BEGIN                                00000990
        Z:=0.0;                                                         00001000
        FOR J:=0 STEP 1 UNTIL Q-1 DO BEGIN                              00001010
          V:=X(I-Q+J);                                                  00001020
```

```
          FOR K:=1 STEP 1 UNTIL J DO                                    00001030
            V:=V-A(J+1,K)*X(I-Q+J-K);                                   00001040
          Z:=Z+A(J+1,J+1)*V**2;                                         00001050
          END J;                                                        00001060
        OUTPUT1(1,'-7Z.2D',Z);                                          00001070
        IF REM(I,5)=0 THEN OUTPUT0(1,'/');                              00001080
        END I;                                                          00001090
      OUTPUT0(1,'//');                                                  00001100
      END CHISQ;                                                        00001110
PROCEDURE WEIGHT(N,P,X,TH,C);                                           00001120
  INTEGER N,P; REAL C; ARRAY X,TH;                                      00001130
  COMMENT CALCULATION OF THE FINAL WEIGHTS USED IN THE CAL-             00001140
          CULATION OF THE ROBUST ESTIMATES TH. THE WEIGHTS              00001150
          ARE MIN(C/(SQRT(Z)*ABS(U)),1) WHERE U IS THE INNOVATION       00001160
          AND Z THE CHI-SQUARE STATISTIC OF THE PRECEEDING P VALUE      00001170
          INPUT:                                                        00001180
            N=NUMBER OF DATA                                            00001190
            P=ORDER OF THE MODEL                                        00001200
            X=DATA VECTOR                                               00001210
            TH=AUTOREGRESSIVE PARAMETER                                 00001220
            C=ROBUSTNESS CONSTANT USED IN THE CALCULATION OF TH.        00001230
          THE WEIGHTS ARE PRINTED DURING THE PROCEDURE,                 00001240
          FOR I=1,..,P EMPTY SPACES ARE PRINTED;                        00001250
  BEGIN INTEGER I,J,K; REAL U,V,W,Z;                                    00001260
      ARRAY A(1:P+1,1:P+2);                                             00001270
    PARCOR(P,TH,A);                                                     00001280
    FOR I:=1 STEP 1 UNTIL P DO SPACE(5);                                00001290
    FOR I:=P+1 STEP 1 UNTIL N DO BEGIN                                  00001300
      U:=X(I);                                                          00001310
      FOR J:=1 STEP 1 UNTIL P DO                                        00001320
        U:=U-TH(J)*X(I-J);                                              00001330
      Z:=0.0;                                                           00001340
      FOR J:=0 STEP 1 UNTIL P-1 DO BEGIN                                00001350
        V:=X(I-P+J);                                                    00001360
        FOR K:=1 STEP 1 UNTIL J DO                                      00001370
          V:=V-A(J+1,K)*X(I-P+J-K);                                     00001380
        Z:=Z+A(J+1,J+1)*V**2;                                           00001390
        END J;                                                          00001400
    W:=SQRT(Z)*ABS(U);                                                  00001410
    IF W<C THEN W:=1 ELSE W:=C/W;                                       00001420
    OUTPUT1(1,'-6Z.3D',W);                                              00001430
    IF REM(I,5)=0 THEN OUTPUT0(1,'/');                                  00001440
    END I;                                                              00001450
  OUTPUT0(1,'//');                                                      00001460
  END WEIGHT;                                                           00001470
FOR I:=1 STEP 1 UNTIL P DO BEGIN                                        00001480
INPUT1(3,'-2Z.3D',TH(I));                                               00001490
OUTPUT1(1,'-2Z.3D',TH(I));                                              00001500
END I;                                                                  00001510
INPUT3(3,'-2Z.3D,-2Z.3D,ZZ.DD/',ETA,SGM,C);                             00001520
OUTPUT3(1,'-5Z.3D,-5Z.3D,ZZ.DD/',ETA,SGM,C);                            00001530
FOR I:=1 STEP 14 UNTIL N DO BEGIN                                       00001540
FOR J:=I STEP 1 UNTIL MIN(I+13,N) DO BEGIN                              00001550
INPUT1(3,'-2ZDD',X(J));                                                 00001560
END J;                                                                  00001570
INPUT0(3,'/');                                                          00001580
END I;                                                                  00001590
FOR I:=1 STEP 5 UNTIL N DO BEGIN                                        00001600
FOR J:=I STEP 1 UNTIL MIN(I+4,N) DO BEGIN                               00001610
OUTPUT2(1,'4ZD,-7ZDB',J,X(J));                                          00001620
```

```
END J;                                                          00001630
OUTPUT0(1,'/');                                                 00001640
END I;                                                          00001650
FOR I:=1 STEP 1 UNTIL N DO BEGIN                                00001660
X(I):=(X(I)-ETA)/SGM;                                           00001670
END I;                                                          00001680
INNOV(N,P,X,TH);                                                00001690
CHISQ(N,P,P,X,TH);                                              00001700
WEIGHT(N,P,X,TH,C);                                             00001710
END;                                                            00001720
END                                                             00001730
```

C. DATA (CHECK 出力例)

```
91   1

.959   911.000     13.300 1.50
 1      842      2      842      3      842      4      910      5      910
 6      910      7      963      8      963      9      963     10      959
11      959     12      959     13      954     14      954     15      954
16      951     17      975     18     1175     19      995     20      987
21     1011     22      980     23      960     24      972     25      985
26      982     27      980     28     1184     29     1184     30      970
31      956     32      958     33      958     34      964     35      949
36      924     37      950     38      922     39      906     40      901
41      894     42      887     43      861     44      861     45      853
46      845     47      864     48      862     49      866     50      861
51      865     52      854     53      852     54      869     55      893
56      910     57      893     58      947     59      932     60      935
61      950     62      945     63      942     64      942     65      942
66      927     67      893     68      888     69      877     70      882
71      862     72      870     73      866     74      869     75      835
76      859     77      814     78      820     79      822     80      824
81      834     82      830     83      821     84      831     85      840
86      852     87      864     88      955     89      982     90      991
91     1000

 -.213      -.213      4.900      -.003
 -.003      3.982       .160       .160      -.140
  .148       .148      -.228       .133       .133
 -.093      1.928     15.235    -12.720      -.343
 2.039     -2.023     -1.291      1.053      1.165
  .003       .068     15.551       .842    -15.249
 -.871       .289       .145       .596      -.964
-1.763      1.995     -1.985     -1.169      -.391
 -.557      -.579     -2.029      -.154      -.756
 -.780      1.225      -.295       .150      -.515
  .147      -.969      -.326      1.096      1.675
 1.223     -1.281      4.005     -1.017       .290
 1.202      -.256      -.121       .096       .096
-1.032     -2.507      -.431      -.898       .271
-1.593       .450      -.427       .087     -2.686
 1.570     -3.544       .152      -.130      -.124
  .484      -.538      -.926       .474       .430
  .683       .720      6.697      2.166       .896
  .923
```

```
 2.16    2.16    2.16     .00
  .00     .00    1.23    1.23    1.23
 1.05    1.05    1.05     .84     .84
  .84     .73    1.86   31.65    3.20
 2.62    4.54    2.16    1.09    1.69
 2.49    2.29    2.16   33.84   33.84
 1.58     .92    1.00    1.00    1.28
  .66     .08     .69     .05     .01
  .05     .13     .26    1.14    1.14
 1.53    1.98    1.00    1.09     .92
 1.14     .96    1.48    1.58     .80
  .15     .00     .15     .59     .20
  .26     .69     .52     .44     .44
  .44     .12     .15     .24     .52
  .38    1.09     .76     .92     .80
 2.62    1.23    4.27    3.76    3.60
 3.44    2.69    2.98    3.68    2.91
 2.29    1.58    1.00     .88    2.29
 2.91

1.000   1.000    .208   1.000
1.000   1.000   1.000   1.000   1.000
1.000   1.000   1.000   1.000   1.000
1.000    .913    .072    .021   1.000
 .454    .348    .790   1.000    .990
1.000   1.000    .066    .306    .017
1.000   1.000   1.000   1.000   1.000
1.000   1.000    .909   1.000   1.000
1.000   1.000   1.000   1.000   1.000
1.000    .871   1.000   1.000   1.000
1.000   1.000   1.000   1.000   1.000
1.000   1.000    .977   1.000   1.000
1.000   1.000   1.000   1.000   1.000
1.000   1.000   1.000   1.000   1.000
1.000   1.000   1.000   1.000    .624
 .590    .382   1.000   1.000   1.000
1.000   1.000    .938   1.000   1.000
1.000   1.000    .224    .739   1.000
 .953
```

>>.