Robust Optimization in League Sports

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As an example of robust optimization, we developed computational algorithms for calculating clinch and elimination statistics in league sports. The clinch (elimination) number is the minimal number of future wins (losses) needed to clinch (to be eliminated from) a specified place in a sports league. The algorithms were applied to play-off spots of Nippon Professional Baseball leagues, and the Kyodo News started distributing some of the result numbers to its member companies during the 2010 season.

Scenario-based modeling

Let L be the set of teams in a league, n the number of teams in L. Suppose we are given the current win-loss records of all teams and the remaining schedule of games. For each team $i \in L$, let w_i and l_i be the current numbers of wins and losses, respectively; the winning percentage is then $w_i/(w_i + l_i)$; and let g_{ij} be the number of remaining games against team $j \in L$. Now, let x_{ij} represent the number of future games team $i \in L$ wins against team $j \in L$. Given a symmetric matrix $g = (g_{ij}) \in \mathbb{Z}^{n\times n}$ with zero diagonals and nonnegative off-diagonals, any matrix $x = (x_{ij}) \in \mathbb{Z}^{n\times n}$ satisfying the conditions

$$\text{(S)} \quad \begin{cases} x_{ij} + x_{ji} \leq g_{ij} & \forall i, j \in L,\ i < j \\ x_{ii} = 0 & \forall i \in L \\ x_{ij} \geq 0 & \forall i, j \in L,\ i \neq j \\ x_{ij} \in \mathbb{Z} & \forall i, j \in L,\ i \neq j \end{cases}$$

represents a possible future scenario, where $\mathbb{Z}$ denotes the set of integers. Let X denote the set of scenarios satisfying (S).

Various leagues can be formulated with this scenario-based model. We confine ourselves to the cases where ties are possible and the winning percentage is used to determine team standings. We also assume that there exists a tie-breaking procedure, without playing additional games, when one or more teams end the season in a tie. Problem structures can be much simpler and easier to solve if the winning point system is used as in football leagues or if ties are not allowed as in Major League Baseball.

Clinching k-th place

Clinching k-th place means that there is no chance of finishing in $(k+1)$-th place or worse even if the team loses all remaining games, where $k = 1, 2, \dots, n-1$. For each team $a \in L$, let us consider the optimization problem

$$\text{(C)} \quad \begin{cases} \max\limits_{\substack{x \in X \\ \alpha \in A_k}} \sum\limits_{j \in L} x_{aj} \\ \text{subject to} \quad x_{aj} + x_{ja} = g_{aj} \quad \forall j \in L \\ \qquad \dfrac{w_a + \sum_{j \in L} x_{aj}}{w_a + l_a + \sum_{j \in L} g_{aj}} \leq \dfrac{w_i + \sum_{j \in L} x_{ij}}{w_i + l_i + \sum_{j \in L} (x_{ij} + x_{ji})} + \alpha_i \\ \qquad\qquad \forall i \in L,\ i \neq a, \end{cases}$$

where A_k is the set of binary vectors satisfying

$$\sum_{\substack{i \in L \\ i \neq a}} \alpha_i = n - k - 1, \quad \alpha_i \in \{0, 1\} \quad \forall i \in L,\ i \neq a.$$

Problem (C) seeks the maximal number of future wins accrued by team a (with the corresponding scenario $\bar{x}$) under the two conditions that no future game of team a ends in a tie and that at least k teams finish with winning percentage better than or equal to that of team a.

Figure 1 shows how the k-th place clinch number of wins is found after solving problem (C).

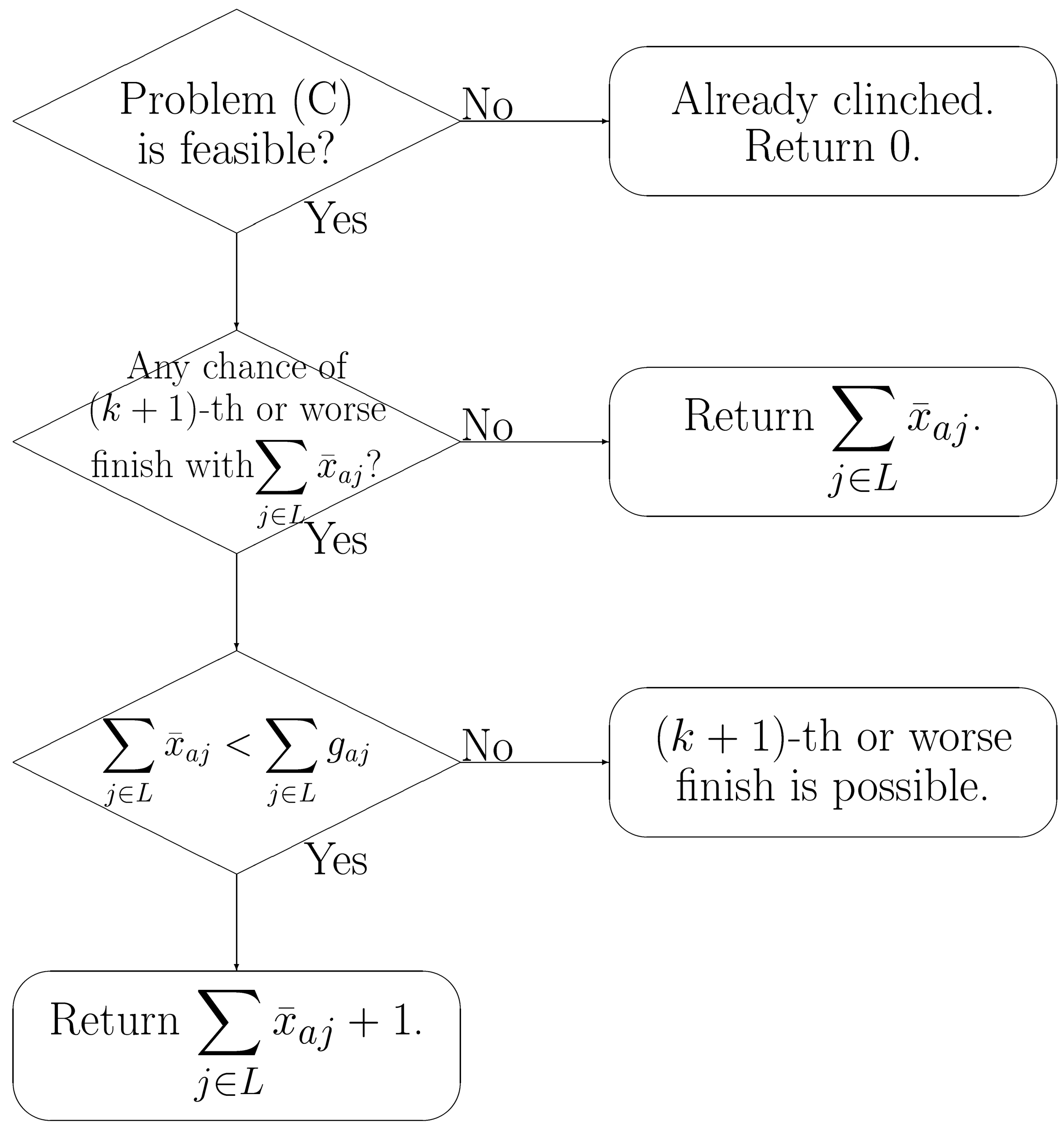

Figure 1. Finding the k-th place clinch number of wins.

When will it actually happen?

The elimination number of losses can be calculated in a similar but dual manner. Being eliminated from k-th place means that there is no chance of finishing in k-th place or better even if the team wins all remaining games, where $k = 1, 2, \dots, n-1$. Finding the minimal number of additional losses for being eliminated from k-th place is equal to finding the maximal number of additional wins for being eliminated from k-th place.

Since the minimal number of additional wins for clinching k-th place is the number in the most unfavorable situations to the team of concern, it is very unlikely that the team will actually need such a number of wins to finish in k-th place or better. Let $\overline{w}$ and $\underline{w}$ be respectively the k-th place clinch and elimination numbers of wins under the circumstances that the team can clinch k-th place if it wins all remaining games and can be eliminated from k-th place if it loses all remaining games. The number of additional wins needed for the team to finish in k-th place or better will lie in the range

$$(1 \leq)\ \underline{w} + 1 \leq w \leq \overline{w} \left(\leq \sum_{j \in L} g_{aj} \right)$$

as shown in Figure 2. Here $\underline{w} + 1$ is the number of additional games that the team must win in order to have any chance of finishing in k-th place or better, and this happens in the most favorable situations to the team.

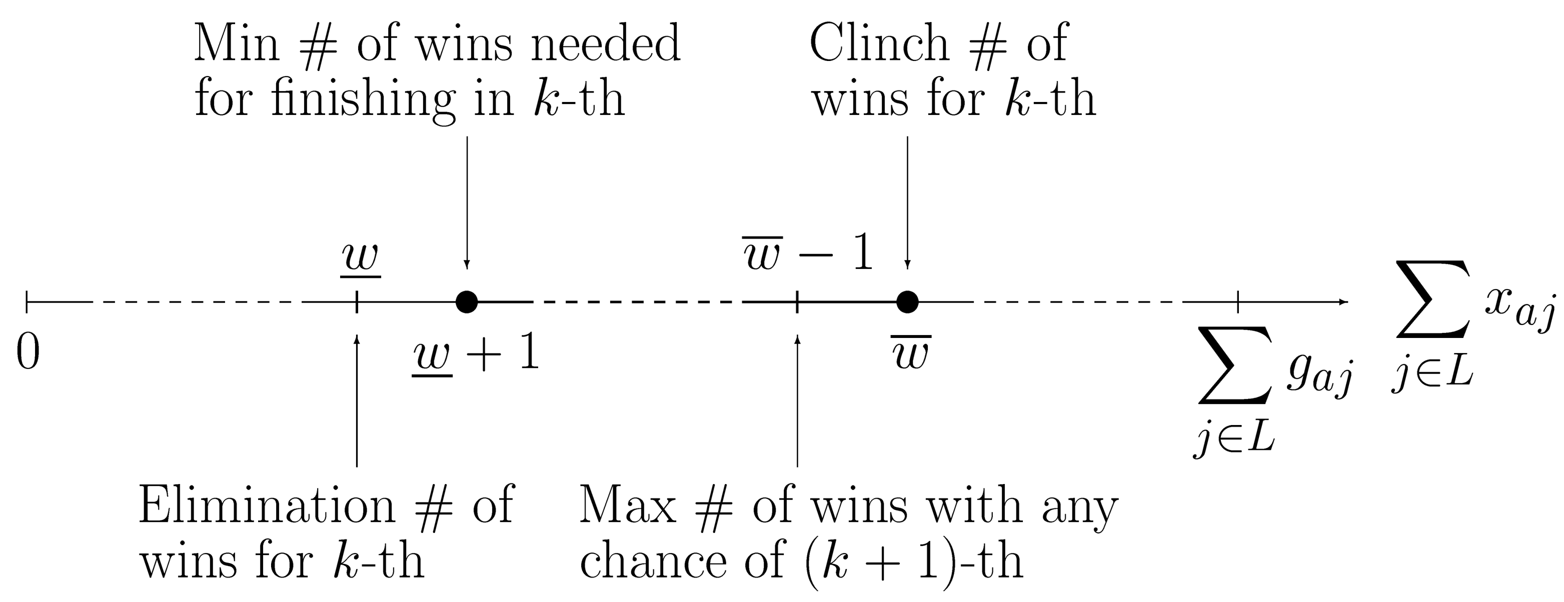

Figure 2. The range of numbers of wins for finishing in k-th place or better.