Discriminant and cluster analysis of possibly high-dimensional time series data by a class of disparities

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For possibly high-dimensional time series data, a basic discriminant statistic has a goodness, and this can be applied to a classification of companies.

Discriminant and cluster analysis of high-dimensional time series data have been an urgent need in more and more academic fields. For possibly high-dimensional and stationary time series data, we show the consistency of classifier under suitable conditions. Also, simulation studies show that that works even in the case of finite observations of training samples. Finally, we conduct the cluster analysis for real financial data. We conclude that our method is suitable for the discriminant and cluster analysis of high-dimensional dependent data.

1 Discriminant analysis

1.1 Setting

\( X = \{X(t) = (X_1(t), \ldots, X_p(t))': t \in \mathbb{Z}\} \quad (p < \infty \text{ or } p \to \infty) \) possibly high-dimensional stationary process with

- mean \( \mu \) and spectral density matrix \( f(\lambda) \).

- Categories:
  
  \[ \pi_1: \mu = \mu^{(1)}, \quad f(\lambda) = f^{(1)}(\lambda), \]
  
  \[ \pi_2: \mu = \mu^{(2)}, \quad f(\lambda) = f^{(2)}(\lambda), \]

Step 1 Independent training samples \( X^{(1)} \) and \( X^{(2)} \) from \( \pi_1 \) and \( \pi_2 \) with size \( n_1 \) and \( n_2 \), respectively, are available.

Step 2 We obtain samples \( X \) with size \( n \).

Classification problem : \( X \in \pi_1 \) or \( \pi_2 \) ?

1.2 Method and result

- To classify \( X \) by

\[ \Gamma(X) = \left( \overline{X} - \frac{X^{(1)} + X^{(2)}}{2} \right)' (\overline{X} - \overline{X}^{(1)}), \]

where

\[ \overline{X} = \frac{1}{n} \sum_{t=1}^{n} X(t), \quad \overline{X}^{(i)} = \frac{1}{n_i} \sum_{t=1}^{n_i} X^{(i)}(t). \]

- Rule:
  
  - \( \Gamma(X) < 0 \to X \) into \( \pi_1 \),
  - \( \Gamma(X) \geq 0 \to X \) into \( \pi_2 \).

- Purpose: consistent classifier is \( P(i|j) \to 0 \) for \( (i, j) = (1, 2), (2, 1) \).

Assumption 1. There exists \( \eta > 0 \) such that

\[ c_1 \eta^2 < \|\mu_1 - \mu_2\| < c_2 \eta^2. \]

Assumption 2. (I) \( p \) is finite and \( n_1, n \to \infty \) \( (n \geq 0) \),

(ii) \( p \to \infty \), and

(i) if \( \eta > 1 \), then both \( n_1 \) and \( n \) are finite or infinite,

(ii) if \( \eta = 1 \), then \( n_1 \to \infty \) and \( n \) is finite or infinite,

(iii) if \( 1/2 < \eta < 1 \), then \( n_1 \to \infty \) and \( n \) is finite or infinite, such that \( p = o(n_1^{1/(1-\eta)}) \),

(iv) if \( \eta = 1/2 \), then \( n_1, n \to \infty \), such that \( p = o(n_1^{1/2}) \),

(v) if \( 0 \leq \eta < 1/2 \), then \( n_1, n \to \infty \), such that \( p = o((n_1^{1/(1-\eta)} + n_2^{1/(1-\eta)})/((n_1^{1/(1-\eta)} + n_2^{1/(1-\eta)})) \).

Theorem 1. Under some appropriate assumptions, \( \Gamma(X) \) is a consistent classifier.

1.3 Simulation

Data: 42 time series with 15 years in a company; Profit and Loss statement, Balance Sheet, and Cash Flow from the first and second section companies of Tokyo Stock Exchange.

Disparity of the cluster analysis:

\[ C(X_1, X_2) = (X_1 - X_2)' (X_1 - X_2). \]

Figure 1: The misclassification rates in simulations (a). The disturbance processes of \( \pi_i, i = 1, 2 \) are AR and ARMA process, respectively.

Figure 2: The misclassification rates in simulations (b). The disturbance processes of \( \pi_i, i = 1, 2 \) are AR and ARMA process, respectively.

Figure 3: Cluster analysis of the first & second sections.